Flood Studies Update

Technical Research Report

Volume I

Rainfall Frequency

Denis L Fitzgerald

Based on Met Éireann Technical Note 61 (Fitzgerald, 2007)

Volume I	**Rainfall Frequency**
Volume II	Flood Frequency Estimation
Volume III	Hydrograph Analysis
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Abstract

This volume describes the FSU method for estimating rainfall frequency at any point in Ireland. The method allows estimation of the rainfall depth of given duration from 15 minutes to 25 days and of desired frequency. The standard unit for rainfall depth is millimetres (mm) and the main measure used to summarise frequency is the return period T in years. The return period is the reciprocal of the annual exceedance probability, which is the probability of a rainfall depth of stated duration being exceeded in any year.

A depth-duration-frequency model is developed which allows estimation of rainfall frequencies for a range of durations for any point location in Ireland. The model comprises an index rainfall (taken as the median of annual maximum values) and a log-logistic growth curve. The growth curve provides a multiplier of the index rainfall.

Gauged records are examined and annual maximum rainfalls are abstracted. The index rainfall is calculated, adjusted, interpolated and mapped on a 2-km grid. The depth-duration-frequency model is constructed in two parts: one exploiting daily data and the other using previously abstracted series of large rainfalls across sub-daily durations. After detailed modelling, checks and adjustments, parameters of the overall depth-duration-frequency model are interpolated and mapped on a 2-km grid. Computer applications then produce gridded outputs of the return-period rainfall depths. An inverse application is developed to allow estimation of rainfall rarity.

An indication is given of the reliability and probable accuracy of the model. The user is instructed to seek up-to-date guidance on the likely effect of projected climate change on extreme rainfalls.

Further information about the research

FSU Technical Research Reports (TRRs) are available in their original form for researchers and practitioners who seek additional information about a method. The original TRRs sometimes document exhaustive application of a method to many catchments. In others, additional options are reported.

Inevitably, the relevance of the original TRRs is influenced by OPW decisions on which methods to implement, and how best to arrange and support them. Readers who consult the original TRRs will notice editorial re-arrangements and compressions, and occasional changes in notation and terminology. These were judged necessary to enhance understanding and use of the FSU methods amongst general practitioners. More significant changes are labelled explicitly as **editorial notes**.

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Parameters of log-logistic distribution

g	Location parameter
a	Scale parameter
c	Curvature parameter

Parameters of rainfall depth-duration frequency model

a, b, e, f	Parameters of the daily part of the DDF model
c_{24}	1-day curvature parameter of DDF model for the 1-day duration [≡ parameter a]
g, h, s	Parameters of the sub-daily part of the DDF model
R(2, 1)	Another name for the index rainfall (≡ $RMED_{1d}$)
$RMED_{1d}$	Index rainfall ≡ 1-day median AM rainfall depth

Symbols

AF	Adjustment factor converting (e.g.) quantile estimates from fixed to sliding duration
c_D	Curvature of growth curve for daily data
c_H	Curvature of growth curve for sub-daily data
D	Duration (mainly in units of days)
f(x)	Probability density function
F(x), F	Cumulative distribution function
K_D	Frequency factor in a version of the rainfall DDF model
ℓn	Natural logarithm
n	Number of years of record, sample size
R	Rainfall depth (mm)
r^2	Coefficient of determination
R(T, D)	T-year rainfall depth (mm) of duration D
$RMED_{1d}$	Median of annual maximum 1-day sliding rainfalls
$RMED_D$	Median of annual maximum D-day rainfalls ≡ R(2, D)
SE_K	Kriging standard error
T	Return period (years)
T_{AM}	Return period on annual maximum scale (years)
T_{POT}	Return period on peaks-over-threshold scale (years)
Var	Variance

Subscripts

1d, 2d, …	1-day, 2-day … sliding-duration depth obtained by converting fixed-duration depth
1d09, 2d09, …	1-day, 2-day … fixed-duration depth
6190	Standard period 1961-90
D, d	Daily
H	Hourly (strictly, sub-daily)

Abbreviations and descriptor names

2p, 3p, 4p	2-parameter, 3-parameter, 4-parameter
AAR_{6190}	Average annual rainfall for standard period 1961-90; denoted by SAAR in FSU
AEP	Annual exceedance probability
AM	Annual maximum
AREA	Catchment area (km^2)
ARF	Areal reduction factor
ARI	Average recurrence interval
C4I	Community Climate Change Consortium for Ireland
CDF	Cumulative distribution function
CEH	Centre for Ecology and Hydrology
Cov	Covariance
DDF	Depth-duration-frequency
DJF	December, January, February
FE	Factorial error
FEH	Flood Estimation Handbook
FSE	Factorial standard error
FSR	Flood Studies Report
FSU	Flood Studies Update
GEV	Generalised Extreme Value
GIS	Geographic information system
JJA	June, July, August
MAM	March, April, May
NERC	(UK) Natural Environment Research Council
NIWA	(NZ) National Institute of Water and Atmospheric Research
NOAA	(US) National Oceanic and Atmospheric Administration
NZ	New Zealand
OPW	Office of Public Works
PDF	Probability density function
POT	Peaks-over-threshold
PWM	Probability-weighted moment
QC	Quality control
SAAR	Standard average annual rainfall – the FSU uses 1961-90 as the standard period
SE, se	Standard error
SON	September, October, November
TBR	Tipping-bucket raingauge/recorder
UK	United Kingdom
US	United States
UTC	Coordinated Universal Time (formerly known as Greenwich Mean/Meridian Time)
Var	Variance

Glossary of terms

Term	Meaning
Annual exceedance probability AEP	Probability of one or more exceedances in a year of a preset rainfall depth (in a given duration)
Annual maximum series	Time series containing the largest value in each year (12-month period) of record for a particular duration
Average annual rainfall	Average annual rainfall (AAR) is evaluated across a particular period. For example, AAR_{6190} relates to 1961-90. The term is synonymous with standard-period average annual rainfall (SAAR).
Average recurrence interval ARI	Average interval (often measured in years) between successive exceedances of a preset rainfall depth (in a given duration)
Calibration	Comparison of a model's predictions with actual data, and adjustment of its parameters to achieve a better fit with reality
Censored sample	Truncated set of data values, e.g. an annual maximum series which (in relevant years) specifies only that the annual maximum was below/above some threshold value
Coefficient of determination r^2	Proportion of variation accounted for by (e.g.) a regression model
Confidence interval	Bounds within which a population parameter is estimated to lie with a stated (usually %) confidence; used to indicate the reliability of an estimate
Convective activity	In meteorological terms, the generation of showers, thunderstorms etc. in air caused to rise by heat transfer from the surface of the earth
Cumulative distribution function CDF	Probability F(x) of a value of the random variable X being less than or equal to x
Curvature parameter	Another name for the shape parameter, a parameter controlling the shape of a distribution; the exponent c in Equation 2.1 is the curvature parameter of the log-logistic distribution, used here to model the rainfall growth curve.
Depth-duration-frequency DDF model	Method of estimating a rainfall amount as a function of duration (D) and frequency; frequency is usually expressed in terms of return period T; the basic components of a DDF model are typically an index rainfall and a growth curve
Dines rainfall recorder	A tilting-siphon rainfall recorder of UK Met Office design that produces a record of rainfall over time
Easting and Northing	Coordinates of a location expressed as distance eastwards and distance northwards from a fixed datum (i.e. reference point)
Fixed-duration rainfall	Rainfall accumulation between fixed hours e.g. the 24-hour total read at 09:00UTC each day
Frontal activity	Rainfall caused by air motions induced by contrasts across zones dividing air masses of differing character
General circulation mode	Computer model of the interactions of the atmosphere with the earth-sun system; used for the prediction of climate change
Geometric mean	n^{th} root of the product of a sample of n values of a positive variable such as rainfall depth
Growth curve	Formula specifying the increase of rainfall (of given duration) with return period; provides the factor by which the index rainfall is multiplied in the depth-duration-frequency model

Term	Meaning
Index rainfall	A chosen reference rainfall depth such as the median of annual maxima. The index rainfall is multiplied by a growth factor in the depth-duration-frequency model
Interpolation	Any method of computing new data points from a set of existing data points
Interquartile range	Measure of dispersion (i.e. variation) defined as the difference between the 3rd quartile and the 1st quartile
Kriging	An interpolation method based on a distance-weighted average of data at neighbouring locations
L-moments	Moments computed from linear combinations of the ordered sample values that lead to summary statistics of (e.g.) variation and skewness and are often more efficient than ordinary moments in parameter estimation of probability distributions; L-moments are intimately related to probability-weighted moments (see also Volume II)
Location parameter	Parameter representing value subtracted from or added to a variable x to translate the graph of its probability distribution along the x-axis
Met Éireann	Irish National Meteorological Service
Ordered sample	Data sample $\{x_{(1)}, x_{(2)}, \ldots, x_{(n)}\}$ in which the elements have been reordered so that $x_{(1)} \leq x_{(2)} \leq \ldots \leq x_{(n)}$.
Peaks-over-threshold (POT) series	For a given duration (e.g. 24 hours), a time series of independent events (abstracted from the period of record) that exceed a preset threshold; the series retains the magnitudes (in mm) and dates of the peak exceedances, together with their times of occurrence; successive POT rainfall events must not overlap
Poisson process	A stochastic process modelling the number of POT event arrivals in a given time interval and/or the inter-event arrival times (i.e. the times between successive independent POT events) as a random variable having an exponential distribution with parameter λ
Probability density function PDF	For a continuous random variable x, the PDF specifies the relative frequency or probability of occurrence of x over all subsets of its range of values
Partial probability-weighted moments	A development of probability-weighted moments used for censored samples
Probability-weighted moments PWMs	Certain weighted linear functions of the ordered sample data that statistical theory shows as both useful and efficient for parameter estimation of probability distributions; L-moments are a development of PWM theory
Quantiles	Values taken at regular intervals from the cumulative distribution function of a continuous random variable; where the CDF is broken into four parts, the quantiles are known as quartiles; if the CDF is broken into 100 parts, the quantiles are known as percentiles
Quartiles	For an ordered sample, the quartiles are the three points that divide the dataset into four equal groups, each group comprising a quarter of the data. The second quartile is the middle observation i.e. the median of the data. The lower quartile is the middle value between the smallest observation and the median, while the third quartile is the middle value between the median and the highest observation. If the quartile (or some other desired quantile) does not correspond to an observation, it is usual to interpolate between successive sample values. [For example, if n is odd, the median corresponds to the middle-ranking value $x_{(i)}$ where i = (n+1)/2; if n is even, the median is taken as the average of $x_{(j)}$ and $x_{(j+1)}$ where j = n/2.]
Residual	Observed value minus the value estimated by a model

Term	Meaning
Return period T	Average number of years between years with rainfalls exceeding a certain value. T is the inverse of the annual exceedance probability. Thus, a 50-year return period corresponds to an AEP of 0.02. The return period is a basic component of the depth-duration-frequency model used to calculate a rainfall depth of the desired frequency.
Return-period rainfall R(T, D)	Estimate of rainfall depth made from depth-duration-frequency model for return period T and duration D, using the relevant model parameters
Scale parameter	Parameter controlling the spread of a distribution
Shape parameter	Parameter controlling the shape of a distribution; referred to in this volume as the curvature parameter
Skewness	A measure of the departure from symmetry of a distribution
Sliding-duration rainfall	Term used for the rainfall total for a given duration when extracting maxima from (effectively) continuous rainfall records; it often exceeds (and cannot be less than) the corresponding fixed-duration rainfall accumulation
Standard-period average annual rainfall SAAR	Standard-period average annual rainfall, i.e. annual average rainfall evaluated across a WMO standard period; in FSU usage, SAAR relates to 1961-90 and is synonymous with AAR_{6190}.
Standard deviation	Measure of dispersion (i.e. variation) of values about their mean
Standard error	Estimated standard deviation of a sample statistic such as the mean, i.e. the standard deviation of the sampling distribution of the statistic
Synoptic station	A meteorological station at which observations are made at standard hours for synoptic (e.g. regional or national) purposes
Tipping-bucket recorder	A type of raingauge which funnels precipitation to one of two balanced buckets of fixed capacity; on reaching capacity, the receiving bucket tips and sends a signal to the recorder; the alternate bucket then becomes the receiver
Unimodal	Having one maximum e.g. on its probability density function

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1 Introduction

1.1 Requirement

The requirement was to develop a gridded set of parameter values summarising the rainfall depth-duration-frequency (DDF) relationship, and to use this to produce consistent estimates of point rainfall frequencies over durations ranging from 15 minutes to 25 days. The estimates were to supersede those provided in Logue (1975) in which the rainfall frequency methods of the UK Flood Studies Report (NERC, 1975) were adapted to Irish conditions.

The requirement for consistency demands that the T-year rainfall depth of duration D, i.e. R(T, D):

- Never exceeds the T-year rainfall depth of a longer duration;
- Differs from grid point to grid point in a manner that is locally and topographically consistent.

1.2 Fixed and sliding durations

The design rainfalls produced by the DDF model are for *sliding durations*. Thus, an 8-day estimate is for 192 consecutive hours and may start at any hour of day. This contrasts with the raw data which are mostly for *fixed durations* e.g. daily values read at 09:00UTC.

1.3 Structure of volume

The volume is principally concerned with the estimation of rainfall frequency at points in Ireland. Chapter 2 outlines the rainfall depth-duration-frequency (DDF) model developed. This is calibrated by reference to maximum rainfall datasets introduced in Chapter 3. Points of important detail include:

- Conversion between different measures of frequency (Section 2.5);
- Quality control of daily and sub-daily rainfall data (Sections 3.1 and 3.2);
- Conversion of rainfall depths from fixed to sliding durations (Section 3.4);
- Methods of spatial interpolation, not least of the index rainfall (in Section 4.2).

Chapter 5 looks at reliability levels and confidence intervals for the gridded rainfall estimates. More technical descriptions of some of these topics are given in appendices or in Fitzgerald (2007). The need to consider the possible effects of climate change is discussed in Chapter 6, before Chapter 7 makes comparisons with the rainfall frequency estimates of Logue (1975).

A final chapter touches on additional matters, not all of which were part of the FSU rainfall frequency research.

1.4 Climate change

An underlying assumption of the statistical analysis presented here is that data from 1941-2004 reasonably represent the upcoming rainfall regime. Given the consensus view that we are in a period of global warming, this is an unsafe assumption: even in the medium term.

General indicators of the effects of global warming on the precipitation regime are available but are heavily dependent on the particular parameterisations used in the general circulation model. Some indications from a 2007 assessment of the Irish climate modelling group (C4I ≡ Community Climate Change Consortium for Ireland) are given. However, appropriate adjustments are not included in the estimates of the return-period rainfalls because it appears that – for quite a number of years yet – indications of the effects of global warming on precipitation regime will change from assessment to assessment. The latest advice on the probable effects of climate change on extreme rainfalls should always be sought.

Figure 1.1 summarises the work-plan followed in the FSU research on rainfall frequency.

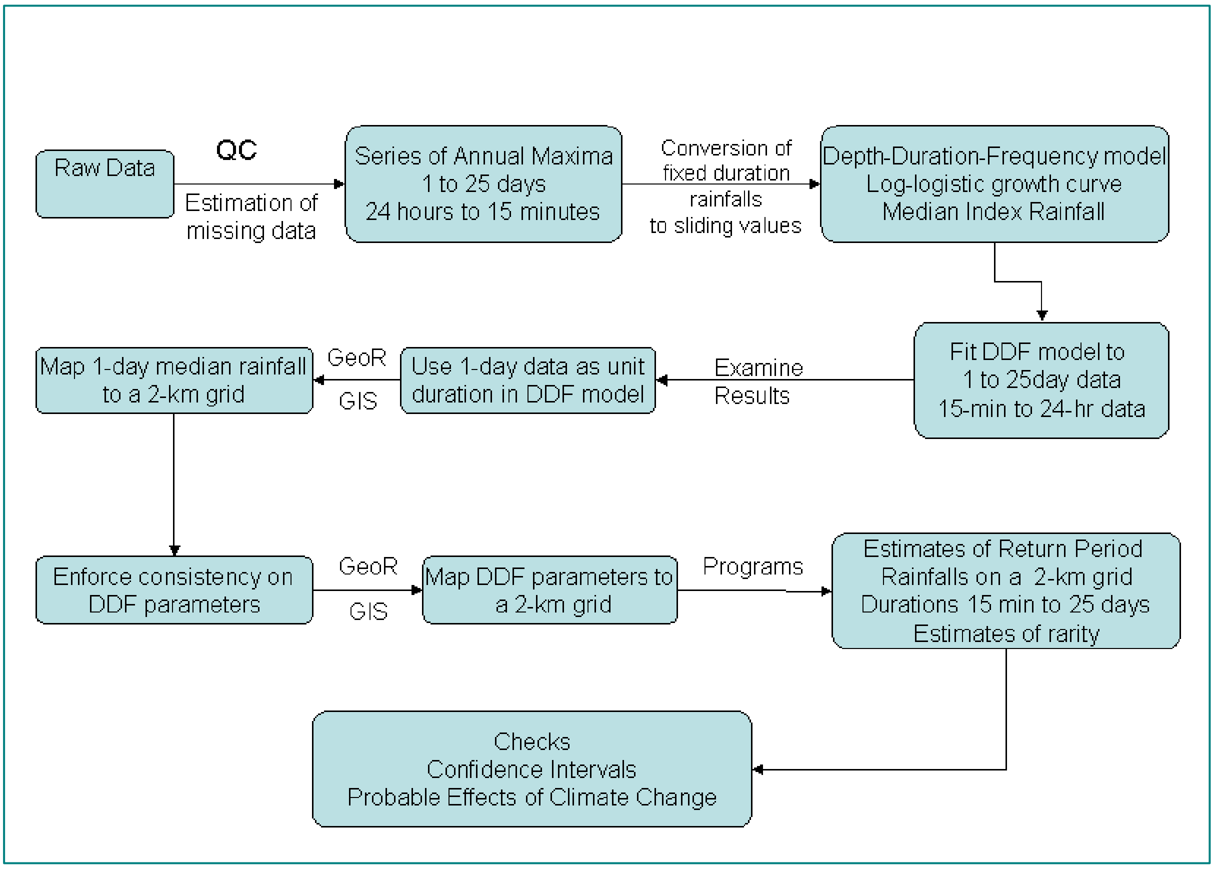


Figure 1.1: Schematic of work-plan

2 The rainfall depth-duration-frequency (DDF) model

The rainfall depth-duration-frequency (DDF) model developed here applies to individual locations. Its mathematical form enables the estimation of rainfall depths for a range of durations (D) and return periods (T).

The DDF model is built around:

- An index rainfall depth;
- A growth curve which provides a multiplier of the index rainfall.

2.1 Growth curve

Let R(T, D) denote the rainfall of duration D and return period T. The log-logistic growth curve takes the form:

$$\frac{R(T,D)}{R(2,D)} = (T-1)^c \quad 2.1$$

where:

$$T = \frac{1}{1-F} \quad 2.2$$

is the return period and F is the cumulative distribution function. R(2, D) corresponds to the index rainfall, taken as the median of the annual maximum rainfalls of duration D. Section A1 of Appendix A explains a particular motivation for using the log-logistic distribution as the growth curve when (as here) the median of the annual maximum (AM) series is adopted as the index rainfall. The approach builds on Fitzgerald (2005).

The return period is the average number of years between *years* with one or more rainfalls exceeding the value R(T, D). Hydrologists refer to this as the return period on the annual maximum scale. Analysis of annual maxima leads naturally to the expression of time intervals in terms of return periods on the AM scale. The reciprocal of return period (i.e. 1/T) is the so-called annual exceedance probability (AEP).

This return period is not quite the same as the average recurrence interval (ARI) between rainfalls exceeding R(T, D), which hydrologists refer to as the return period on the peaks-over-threshold scale. Langbein's formula interconnects the two measures (see Section 2.5.4).

2.2 Annual maximum rainfall data

Some features of the annual maximum (AM) series used in the study are introduced in Section A2. The chief point to note is that the modelling treats AM series from daily rainfall data separately from those of sub-daily rainfall data.

2.3 Index rainfall

The index rainfall adopted in the FSU is the median of the AM rainfalls of the relevant duration, RMED. Precisely half of annual maxima exceed the median. Thus, RMED corresponds to F = 0.5 and (from Equation 2.2) T = 2 years. The median AM value is also adopted as the index variable in flood frequency estimation (see Volume II), where the notation QMED is used.

RMED varies according to the duration of rainfall being considered. Rather than adopting the notation $RMED_D$, it proves convenient to label the index rainfall as R(2, D).

A plausible form for the variation of the index rainfall with duration is (see Section A3):

$$R(2, D) = R(2, 1)\, D^s \qquad 2.3$$

where D = 1 is a suitably chosen unit duration. The unit adopted here is one day. R(2, 1) is the median AM 1-day rainfall and plays a pivotal role in the DDF model.

It should be noted in particular that the index rainfall adopted in the final model is taken as the median AM rainfall depth for sliding maxima of 24-hour duration. It is nevertheless referred to as the 2-year 1-day rainfall.

The median AM D-day rainfall, R(2, D) increases as duration increases above one day and reduces as duration reduces below 24 hours. The exponent s determines the scaling effect and is referred to as the *scale parameter*.

2.4 Structure of DDF model

2.4.1 Overall structure

The depth-duration-frequency (DDF) model combines Equations 2.3 and 2.1 to give:

$$R(T, D) = R(2, 1)\, D^s\, (T - 1)^c \qquad 2.4$$

The exponent c (introduced in Equation 2.1) is the curvature parameter of the log-logistic growth curve (see Section A1). It determines the multiplier of R(2, D) which yields R(T, D), i.e. the rainfall depth of return period T and duration D.

The exploratory data analysis reported in Section A3 provides some further insight into the modelling. The final form of the DDF model uses Equation 2.4, with the exponents c and s specified by sub-models. These are now summarised.

2.4.2 Model exponents for durations of 1 to 25 days

The daily part of the model is:

Curvature parameter: $c_D = a + b\, \ell n\, D$ 2.5

Scale parameter: $s_D = e + f\, \ell n\, D$ 2.6

Here, ℓn denotes the natural logarithm and the subscript D denotes daily. The background to these sub-models is discussed in Section A4.2. Note also the discussion of parameters in Section 2.4.4.

Box 2.1: Editorial notes on the rainfall DDF model and its parameters

The daily part of the model has four parameters: a, b, e and f. The sub-daily part of the DDF model has three parameters: g, h and s. In addition, the index rainfall – the median AM 1-day rainfall, R(2, 1) – forms an integral part of the model.

The index rainfall is determined ahead of calibration of the daily part of the model which, in turn, is undertaken before the sub-daily modelling is concluded.

Note that the sub-daily parameter g is fixed by the daily part of the model, so that g ≡ a. This identity is needed to make the daily and sub-daily parts of the model "meet" at D = 1 day. Thus, the value of g in Equation 2.7 is identical to the value of a in Equation 2.5.

The sub-daily modelling is especially intricate. Some aspects are treated in Appendix C, with Fitzgerald (2007) providing additional details. Parameters h and s relate only to the sub-daily part of the DDF model. After exploratory analysis, it transpires that these two parameters are fixed by a specific set of rules. Thus, only the index rainfall and the four parameters of the daily part of the model have to be mapped in Chapter 4 to generalise the DDF model across Ireland.

Even with the further details presented in Section A1, the model is not easy to understand. The DDF model incorporates explicit fits of the log-logistic distribution to AM rainfalls of an individual duration. In this latter usage, the log-logistic has three parameters:

- A location parameter, g;
- A scaling parameter, a;
- A curvature (or shape) parameter, c.

Importantly, the focus of research is the rainfall DDF model itself. Fitzgerald refers to this as a *portmanteau model* because it bridges across all durations. Regrettably, it is all too easy to confuse parameters of the log-logistic distribution with parameters of the rainfall DDF model.

In part to respect these subtleties, Fitzgerald (2007) uses multiple annotations for some parameters. In the final model, the parameters a and g are identical. However, each of these is referred to by a further alternate name: the parameter a as c_1 and the parameter g as c_{24}, reflecting that the parameter corresponds to the implied curvature of the growth curve at the 1-day duration in the daily part of the model and at a duration of 24 hours in the sub-daily part of the model. In the final DDF model, all these parameters take the same value, so that $c_1 \equiv a \equiv g \equiv c_{24}$.

2.4.3 Model exponents for durations of 24 hours to 15 minutes

The sub-daily part of the model is:

Curvature parameter: $c_H = g + h \, \ell n \, D$ 2.7

Scale parameter: $s_H = s$ 2.8

[**Editorial notes**: The subscript H appears to derive from hourly. Use of the subscript S-D for sub-daily would have been a more precise qualifier. The curvature parameter c is again a function of duration but the scale parameter s is not. The formulation was influenced by the exploratory data analysis reported in Section A3. Further details of the sub-daily part of the DDF model are given in Box 2.1.]

2.4.4 Link between daily and sub-daily parts of the model

Many aspects of the research are highly technical. A particular difficulty lies in understanding the link between daily and sub-daily parts of the rainfall DDF model and its parameters (see Box 2.1). It should be noted that D is measured in days in both parts of the model. Thus, a duration of 12 hours is represented in Equation 2.7 by a value of D = 0.5.

2.5 Link between return periods on the AM and POT scales

2.5.1 Annual maximum and peaks-over-threshold series

By definition, the annual maximum (AM) series comprises the highest fall in each year. The second highest fall in a year is ignored, whether or not it exceeds the highest fall in other years. In contrast, the peaks-over-threshold (POT) series consists of all rainfalls exceeding a certain threshold together with their times of occurrence. An alternate name for the POT series is the partial duration series.

2.5.2 Return period

The return period T is best thought of as the inverse of the annual exceedance probability. For example, the rainfall corresponding to T = 50 has a probability of 0.02 of being exceeded in any year. It can be helpful to refer to this return period as the return period *on the annual maximum scale*, to avoid possible confusion with other measures of frequency.

Though it is often misunderstood by the public, return period is a useful concept to the professional. Its importance is perhaps best conveyed in understanding the often appreciable risk of an extreme event being experienced in the long run. The risk equation (see Equation 2.9 in Box 2.2) expresses the chance of the T-year event being exceeded in a long run or lifetime of L years.

2.5.3 Average recurrence interval (ARI)

The analysis of POT series gives the average interval between rainfall events that exceed a particular value. This is often termed the *average recurrence interval* (ARI) for a given duration D. For high values of T, values of ARI and T are nearly equal. But for T less than 20 years the difference is large enough to be important. Hydrologists often refer to ARI as the return period on the *peaks-over-threshold* scale, and denote it by T_{POT}.

Box 2.2: The risk equation

The probability of the *T*-year extreme event being exceeded at least once in L years is:

$$r = 1 - (1 - \frac{1}{T})^L \qquad 2.9$$

Hydrologists typically refer to this as *the risk equation*. It can be reasoned by rewriting:

$$1 - r = (1 - \frac{1}{T})^L$$

and noting that the probability of the T-year extreme *not* being exceeded in L years (i.e. the left-hand side of the equation) is simply the probability of the T-year extreme *not* being exceeded in any of the L individual years (i.e. the right-hand side of the equation).

Risks are not always as one imagines. For example, there is an even chance (r = 0.5) that the 100-year rainfall is exceeded in any 69-year period:

$$1 - (1 - \frac{1}{100})^{69} \approx 0.500$$

The risk equation can be applied to flood peaks as well as to extreme rainfall depths of a given duration. It should be noted that such applications assume that the system producing the extremes is stationary, i.e. that annul maxima are statistically independent and drawn from the same underlying distribution. This assumption is compromised by climate change. In the case of flood risk, it may also be compromised by catchment change.

2.5.4 Langbein's formula

Langbein (1949) provides a formula relating the two measures of frequency, i.e. T and ARI:

$$\frac{1}{T} = 1 - \exp\left(-\frac{1}{ARI}\right) \qquad 2.10$$

This yields pairs of values such as:

- T = 1.16 years when ARI = 0.5 years (twice per year frequency);
- T = 1.58 years when ARI = 1 year (once per year frequency);
- T = 2.54 years when ARI = 2 years (one in two years frequency).

For return periods of five years or longer, the approximation T = ARI + 0.5 suffices.

This research principally uses AM rainfall series. Thus, the rainfall DDF model developed is for return-period rainfall depths R(T, D). However, Langbein's formula is nevertheless very useful for converting any ARI of interest into an equivalent return period or *vice versa*.

For those favouring the notation T_{AM} and T_{POT}, Langbein's formula can be written:

$$\frac{1}{T_{AM}} = 1 - \exp\left(-\frac{1}{T_{POT}}\right) \qquad 2.11$$

3 Rainfall data for the FSU

3.1 Data selection and quality control – daily data

The requirement was for annual maximum (AM) series for at least six durations ranging from one to 25 days. On assessing the quality control needed to generate and validate each such AM series, it was decided that prior production of complete daily rainfall series was the better option.

The study therefore centred on daily data taken from the Met Éireann archive for which periods within 1941-2004 had already undergone extensive quality control. As a precaution, rainless months and daily rainfalls in excess of 75.0 mm were re-examined. Some faulty data were corrected before forming the initial table of daily rainfalls.

Stations of high quality were judged from the small number of accumulated readings (i.e. observations representing more than one day's rain) and the small number of missing days. Any accumulated readings were disaggregated (i.e. apportioned) into daily values by reference to data from neighbouring stations. Where necessary, observations on missing days were estimated using data from neighbouring stations. The data infill methods are detailed in Appendix B.

Use of the infill methods yielded complete series of daily rainfall depths for 474 stations. Their lengths of record ranged from 20 to 64 years: 64 years being the maximum possible when using data drawn from 1941-2004. The mean period of record was 41.22 years.

Using these complete daily series, annual maxima were extracted for 11 durations: 1, 2, 3, 4, 6, 8, 10, 12, 16, 20 and 25 days. The AM values (and their starting dates) were loaded into a table of 214,918 rows ($\approx$ 474 x 11 x 41.22). The 474 stations are shown in Map 3.1a, where 103 stations from Northern Ireland are also mapped.

3.2 Data selection and quality control – sub-daily data

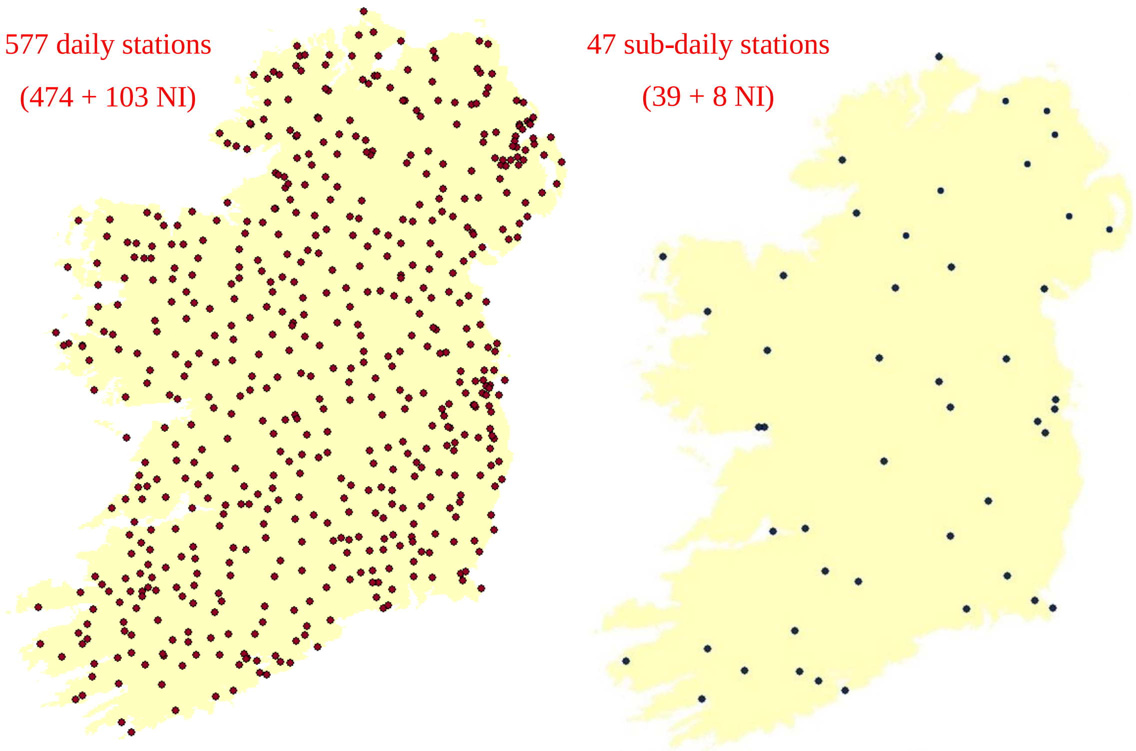
3.2.1 Data selection

Data for nine (sliding) durations between 15 minutes and 24 hours were available from 39 stations for record lengths ranging from 15 to 55 years. All but two of the 39 stations have 30 or more years of record. Their locations are shown in Map 3.1b, where eight stations from Northern Ireland are also mapped.

Most of the short-duration data derive from Dines rainfall-recorder charts. From about the mid-1990s, tipping-bucket recorders (TBRs) began to replace some of the Dines recorders. However, in the periods of overlap, the differences between readings from Dines recorders and TBRs were found to be generally small. No adjustment for the transition was required.

Maximum rainfalls attaining or exceeding at least one of a set of thresholds were extracted. The thresholds used in the archive of maximum rainfalls are shown in Table 3.1. The date assigned to the fall is the day on which most, or all, of the total for the rainfall event was recorded.

(a) Raingauges in daily dataset ***(b) Raingauges in sub-daily dataset***



Map 3.1: Raingauges used in rainfall frequency research

Table 3.1: Thresholds used to identify maximum rainfalls from sub-daily data

Duration	15 mins	30 mins	1 hour	2 hours	3 hours	4 hours	6 hours	12 hours	24 hours
Threshold (mm)	4.0	5.0	6.0	10.0	12.5	15.0	20.0	25.0	30.0

3.2.2 Quality control

Quality control consisted of:

- Checking the dates of occurrence;
- Checking the 24-hour maximum rainfalls of 30 mm or more against 09:00-09:00UTC totals;
- Checking the tabulations for consistency across the different durations.

Doubtful or missing values were estimated by reference to nearest available stations.

Sixteen of the 39 sites correspond to synoptic stations, allowing a more extensive scrutiny of the maximum rainfalls. The tabulated values – which are absolute maximum rainfalls (i.e. for sliding sub-daily durations) – were cross-checked for consistency with maximum rainfalls based on clock-hour rainfall data. Checks were made both of the amounts and their dates.

The requirement for annual maximum (AM) series led to anticipation of difficulties caused by annual maxima falling short of the Table 3.1 thresholds used to abstract the maximum rainfalls. In the case of the synoptic stations, relatively good estimates could have been derived from clock-hour values. However, the strategy adopted for all stations was to trust in the general good quality of the data and assume that any missing maximum rainfalls were below the relevant threshold and unimportant to the model-fitting.

The intention had been to regard the AM series as censored at the threshold. Indeed, special methods were developed to accommodate this feature (see Fitzgerald, 2007). However, it transpired (see Section A3.4) that the method finally adopted for fitting the rainfall DDF model uses only the upper half of the AM data, i.e. those ordered AM values greater than or equal to the sample median. It also transpired that these subsets of the AM series were already complete at each of the 39 sub-daily stations.

3.3 Data for Northern Ireland

With agreement from the UK Met Office, the Centre for Ecology and Hydrology supplied selected daily and hourly AM series for Northern Ireland from the Flood Estimation Handbook research (Faulkner, 1999). The data used here are:

- AM data for 103 daily stations at durations of 1, 2, 4 and 8 days (these series had at least 20 years of record);
- AM data for eight sub-daily stations at durations of 1, 2, 4, 6, 12, 18 and 24 hours (these series had lengths of record between 11 and 19 years).

3.4 Conversion from fixed to sliding durations

The data available were 1, 2, 3, 4, 6, 8, 10, 12, 16, 20 and 25-day annual maximum depths derived from daily totals read at 09:00UTC. However, the chief application of the rainfall DDF model is to estimate return-period rainfalls for so-called sliding durations (see Section 1.2). How to do this from the 09:00-09:00UTC data involves adjustment from fixed to sliding durations.

The matter was examined in two ways. In both cases, the investigation used clock-hour data from 14 longer-term synoptic stations. It should be noted that no allowance was made for the further transition from clock-hour values to absolute values. Inspection showed that, even for 24 hours, such differences were very small. In other words, the maximum 24-hour total based on clock-hour data was found to be very close to the maximum 24-hour total based on 15-minute data or on TBR data.

A further point of detail was adoption of April-March as the "rainfall year". Use of the calendar year led to noticeable end-of-year effects – especially at the longer durations – through the neglect of some important large rainfalls spanning the end of December and the beginning of January. The sensitivity of the return-period rainfall estimates to end-of-year effects was reduced by adopting April-March as the rainfall year.

3.4.1 Ratios of quantiles

The first approach looked at the ratios of quantiles for sliding-duration as opposed to fixed-duration annual maxima. For 14 longer-term synoptic stations, the log-logistic distribution was fitted to the 1, 2, 3, 4, 6, 8, 10, 12, 16, 20 and 25-day annual maxima of the 09:00-09:00UTC rainfall totals and also to 24, 48, 72, 96, 144, 192, 240, 288, 384, 480 and 600 clock-hour annual maximum depths.

Using the fitted distributions, return-period rainfalls of 2, 5, 10, 20, 50, 100, 250, 500 and 1000 years were evaluated and their ratios examined. Summary results are presented in the main part of Table 3.2. The final column is discussed later.

The ratio of sliding-duration to fixed-duration quantiles is found to be about 1.15 for the 1-day duration (first row of results in Table 3.2). As is to be expected, the ratios are closer to 1.00 at longer durations. Nevertheless, the effect remains detectable at durations of 20 days.

The values in the main part of Table 3.2 are mean experimental values of the ratio of sliding-duration to fixed-duration quantiles based on data from 14 longer-term synoptic stations. They are candidate adjustment factors for converting from fixed to sliding durations.

Table 3.2: Mean adjustment factors (fixed to sliding durations) – 14 synoptic stations

D (duration in days)	Quantile, T (return period in years)									Events
	2	5	10	20	50	100	200	500	1000	
1	1.153	1.148	1.147	1.146	1.145	1.146	1.148	1.149	1.152	1.110
2	1.076	1.064	1.058	1.051	1.044	1.038	1.032	1.026	1.022	1.060
3	1.062	1.048	1.040	1.032	1.023	1.017	1.009	1.002	0.996	1.044
4	1.044	1.036	1.032	1.028	1.023	1.020	1.016	1.013	1.011	1.037
6	1.041	1.029	1.022	1.015	1.007	1.002	0.994	0.989	0.984	1.027
8	1.042	1.032	1.026	1.021	1.015	1.010	1.004	1.000	0.996	1.024
10	1.034	1.027	1.022	1.018	1.013	1.010	1.005	1.002	0.999	1.021
12	1.026	1.031	1.034	1.037	1.040	1.043	1.048	1.051	1.054	1.017
16	1.023	1.022	1.021	1.021	1.021	1.021	1.021	1.021	1.021	1.014
20	1.017	1.015	1.015	1.014	1.013	1.012	1.012	1.011	1.011	1.011
25	1.016	1.011	1.008	1.006	1.003	1.001	0.999	0.995	0.993	1.010

3.4.2 Ratios of event depths

The second approach began by setting thresholds of 23, 30, 35, 40, 48, 55, 60, 65, 70, 75 and 80 mm (respectively) to the maximum rainfalls noted (from the 09:00-09:00UTC data) for durations of 1, 2, 3, 4, 6, 8, 10, 12, 16, 20 and 25-days. These D-day exceedances were compared with the corresponding clock-hour values. The correspondence (between daily and hourly representations of the events) was achieved by ensuring that the start time (from hourly data) was within a certain interval of the inferred mid-point of the D-day event (from daily data).

Having found an acceptable match (amongst the hourly data) to the D-day events identified from 09:00-09:00UTC data, mean ratios of the adjustment factor were calculated as:

$$AF = \frac{1}{n}\sum_{i=1}^{n}\frac{x_i}{y_i}$$

where x_i denotes the rainfall depth of the i^{th} event based on hourly data, and y_i denotes the depth based on daily data. This method of averaging contrasts with that used by Dwyer and Reed (1995), who favoured the ratio of the mean value of x to the mean value of y, thereby giving greater weight to the larger events.

Values of the adjustment factor from events are shown in the final column of Table 3.2. Because they have been derived from the full set of annual maximum D-day exceedances, it is reasonable to compare them with the 2-year values (which correspond to the median AM event).

3.4.3 Comparison of mean adjustment factors by the two approaches

The mean event-based adjustment factors are seen to be smaller than those based on the ratio of quantiles. The difference is especially marked at the 1-day duration (see also Figure 3.1). [**Editorial note**: The finding can be explained as follows. The selection of annual maximum events from daily rainfall data favours some 24-hour rainfall events that are particularly well synchronised with the 09:00-09:00UTC measurement day. In consequence, in some years of record, the dates of AM 1-day events (abstracted from daily data) do not match the dates of AM 24-hour events (abstracted from hourly data). This mismatch makes the adjustment factors in the final column of Table 3.2 – which are explicitly derived from *matched* event depths – unsuitable for converting rainfall quantiles from fixed to sliding durations. The appropriate conversion factors – highlighted in red in Table 3.2 – are those derived as the ratios of quantiles.]

3.4.4 Variability of adjustment factors

Table 3.2 reports a mean adjustment factor of 1.153 for the 2-year 1-day case. The spread of results across the 14 stations was from 1.086 to 1.257, with a mean of 1.153, a median of 1.148 and a standard deviation of about 0.03. These adjustment factors have been explicitly derived as the ratios of sliding-duration to fixed-duration rainfalls.

The ratios decrease with return period in some cases but are nearly constant for 1-day rainfalls and for durations of 16 days and longer. As expected, the ratios decrease with duration (see different rows in Table 3.2) in a reasonably regular manner.

Because of its key role as the index rainfall, the ratios for the 2-year quantile are of greatest importance. Moreover, the conversion from 1-day 09:00-09:00UTC to 24-hour median values is crucial to unifying the daily and sub-daily parts of the overall rainfall DDF model.

In a further check, the medians of the AM series at the 39 stations for which absolute 24-hour maxima were available were compared with the medians of the 09:00-09:00UTC annual maximum series *over the same years*. Some summary statistics are presented in Table 3.3.

Table 3.3: Adjustment factor for 2-year 1-day rainfall – across 39 sub-daily stations

Dataset	Minimum	Q1	Median	Q3	Maximum	Mean	Standard deviation
39 sub-daily stations	1.000	1.091	1.123	1.164	1.331	1.124	≈0.05

3.4.5 Appraisal

The adjustment factors based on the ratio of quantiles (Section 3.4.1) are more relevant than those based on events (Section 3.4.2). For 1-day rainfalls, the mean (1.153) and median (1.148) adjustment factors both round to 1.15. This value was therefore adopted for general use for AM 1-day rainfalls, noting that it is intermediate to the results of detailed studies by Dwyer and Reed (1995) and NIWA (2002), which yielded 1-day adjustment factors of 1.16 and 1.14 respectively. The 2-year quantile-based results from Table 3.2 were similarly adopted at other durations, after rounding to 2 decimal places (see Table 3.4).

Table 3.4: Adjustment factors adopted for AM D-day rainfalls

Duration, D (days)	1	2	3	4	6	8	10	12	16	20	25
Adjustment factor	1.15	1.08	1.06	1.05	1.04	1.04	1.03	1.03	1.02	1.02	1.01

The adjustment factors are summarised in Figure 3.1.

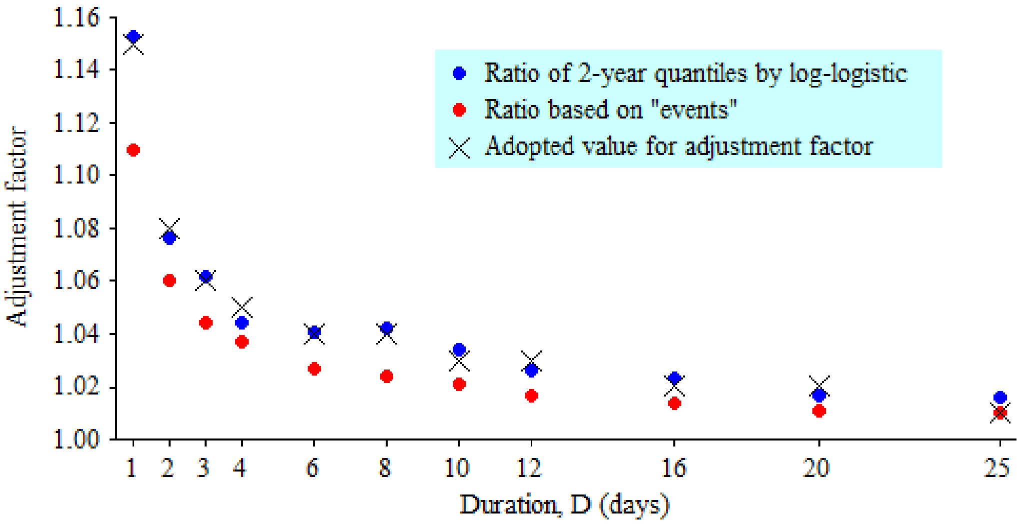


Figure 3.1: Adjustment factors for conversion from fixed to sliding durations

3.5 Notation

Fixed-duration rainfalls are labelled 1d09, 2d09, …, 25d09, to denote durations of 1, 2 … 25 rainfall-measurement days ending at 09:00UTC. Sliding-duration rainfalls obtained by multiplying the fixed-duration rainfalls by the appropriate conversion factor are referred to as 1d, 2d, 3d, …, 25d rainfalls. Sub-daily durations are always treated as sliding durations. The Irish sub-daily data are absolute maxima extracted from rainfall events for durations up to and including 24 hours. The 1d value – obtained by converting fixed-duration depths to sliding-duration depths – provides a close approximation to these values and has the advantage of being much more widely available.

4 Implementing the DDF model

4.1 Requirement

Two applications have to be supported at each 2-km grid point. These are estimating the:

- Design rainfall depth R(T, D), for given return period T and duration D;
- Rainfall rarity T, for given rainfall amount R in duration D.

The implementation builds on two sets of mapping work:

- Mapping the index rainfall (Section 4.2);
- Mapping the other parameters of the rainfall DDF model (Section 4.3).

4.2 Mapping the index rainfall: the median AM 1-day rainfall

4.2.1 Introduction

The pivotal value in the rainfall DDF model is the median AM 1-day rainfall, $RMED_{1d}$. From the research reported in Section 3.4, this is closely approximated by:

$$RMED_{1d} = 1.15\ RMED_{1d09} \qquad 4.1$$

Some 577 values of $RMED_{1d}$ were available, with 103 of them in Northern Ireland (see Map 3.1a). This network was judged rather too sparse to interpolate directly to map values of $RMED_{1d}$ on a 2-km grid (some 21000 points in all). A method of reinforcement was therefore developed.

4.2.2 Linking $RMED_{1d}$ to SAAR

The density of the network was strengthened by exploiting a map of 1961-90 average annual rainfall (Fitzgerald and Forrestal, 1996) based on extensive scrutiny of gauged data and informed climatological interpretation. A very strong linear relationship was found, allowing $RMED_{1d}$ to be estimated from SAAR and grid location:

$$RMED_{1d} = 0.03331\ SAAR + 0.000062\ Easting - 0.0000353\ Northing \qquad 4.2$$

Here Easting and Northing are grid references in metres, and $RMED_{1d}$ and SAAR are (as usual) in mm. Equation 4.2 has a coefficient of determination (r^2) of 0.993.

4.2.3 Gridding SAAR

To exploit Equation 4.2, values of SAAR are needed on the 2-km grid. The stations for which SAAR was available numbered 946, of which 242 were in Northern Ireland. Drawing on accumulated experience in estimating and mapping average annual rainfall (e.g. Fitzgerald and Forrestal, 1996), 101 values were added in data-sparse areas.

The resulting 1047 data points were used to map SAAR on to the 2-km grid by applying geostatistical methods (Kitanidis, 1997) and the R package *geoR* (Ribeiro and Diggle, 2001):

- The gross dependence of annual average rainfall on elevation and location was first removed by linear regression [model not reported here];
- The residuals from the model were interpolated to the grid by ordinary kriging with a moving neighbourhood, i.e. using weighted linear combinations of nearby values;
- Grid-point values of SAAR were formed by adding the gridded residuals to the values of SAAR modelled (from elevation and location) by regression.

Comparison with previously mapped values of AAR_{6190} (e.g. Fitzgerald and Forrestal, 1996) showed very good agreement.

4.2.4 Gridding $RMED_{1d}$

Kriging was used to grid $RMED_{1d}$ in a similar fashion. The number of stations having values of both $RMED_{1d}$ and SAAR was 468. The residuals from the Equation 4.2 regression of $RMED_{1d}$ on SAAR, Easting and Northing were interpolated to the 2-km grid. The grid-point values of $RMED_{1d}$ were then formed by adding the gridded residuals to the values of $RMED_{1d}$ modelled by Equation 4.2. The final map of the index rainfall is shown in Map 4.1.

4.2.5 Reliability of interpolated values of the index rainfall

A measure of the uncertainty in the final map was gained by studying the 109 (i.e. 577 – 468) stations for which a value of $RMED_{1d}$ had been obtained in Chapter 3 but had not been used in the mapping (because they lacked a gauged value of SAAR). Using gridded $RMED_{1d}$ as the data values, the 109 kriged estimates of $RMED_{1d}$ were obtained together with their kriging standard errors (SE_K).

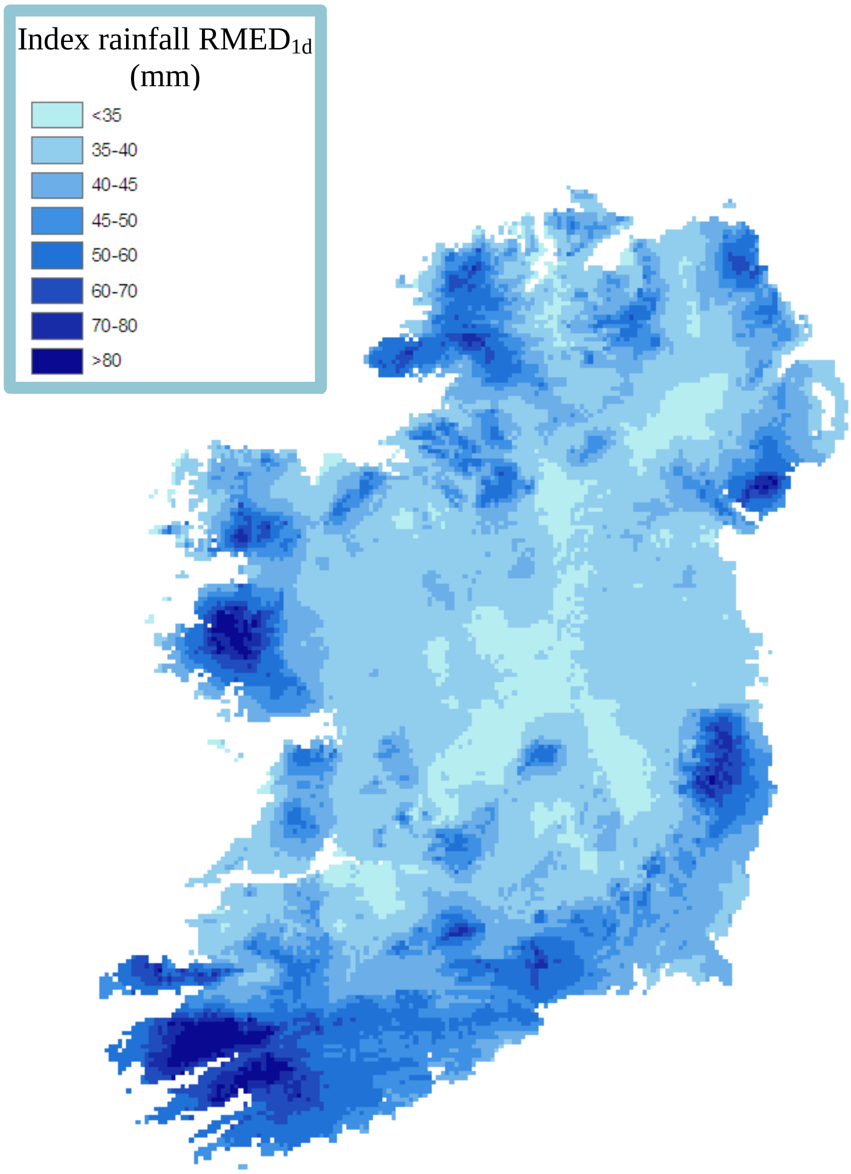
Denoting the kriged estimate by $RMED_{1d,K}$, the factorial error in $RMED_{1d}$ is given by:

$$FE = \frac{RMED_{1d}}{RMED_{1d,K}} \qquad 4.3$$

FE was found to be approximately normally distributed across the 109 stations, with a mean value of 0.995 and a standard deviation of 0.084. Thus, the factorial standard error in $RMED_{1d}$ is assessed as:

$$FSE = e^{0.084} = 1.09 \qquad 4.4$$

[**Editorial note**: Thus, about 68% of estimates are expected to lie within the factorial range 1/1.09 to 1.09 of the gauged value of $RMED_{1d}$. Fitzgerald (2007) broadly confirms this by noting that 66 out of the 109 cases (i.e. 61% of estimates) yield differences less than 7.5%. This degree of model performance is something that hydrologists can only dream of. The equivalent FSE for estimation of QMED from physical catchment descriptors (see Chapter 2 of Volume II) is 1.37, making confidence intervals about four times wider.]



Map 4.1: Final map of the index rainfall $RMED_{1d}$ i.e. R(2, 1)

This independent test of the interpolation justifies the assumption that the mapped value is a good estimator of the actual median rainfall at a site.

The kriging standard error, SE_K, of $RMED_{1d,K}$ reflects the variability of the estimates which are weighted means of the surrounding grid-point values. The standard error will be higher where the median rainfall changes more rapidly with distance, e.g. in/near mountainous areas. The kriging standard error was found to be reasonably well approximated by:

$$SE_K = 0.001\ RMED_{1d,K}\ d^2 \qquad 4.5$$

with a coefficient of determination (r^2) of 0.81. [**Editorial note**: Here, d denotes distance; the unit is thought to be km.] SE_K increases monotonically with $RMED_{1d,K}$ and ranges from about 1 mm at the minimum mapped $RMED_{1d,K}$ value of 31 mm to about 9 mm at the maximum mapped $RMED_{1d,K}$ value of 94 mm.

As the higher values of $RMED_{1d,K}$ occur in the mountains – where the density of the raingauge network is typically lowest – the uncertainty attaching to the interpolated values is greatest there. The kriging standard error provides an indication of the reliability of $RMED_{1d,K}$.

4.3 Mapping the parameters of the DDF model

4.3.1 Durations of one to 25 days

No useful relationships were found between the four parameters of the daily part of the DDF model – a, b, e and f – and the median AM rainfall, SAAR, Easting or Northing. Hence, all parameters were interpolated to the 2-km grid using ordinary kriging with a moving neighbourhood, i.e. based on distance-weighted averaging.

4.3.2 Durations of 24 hours to 15 minutes

The structure adopted for the sub-daily part of the DDF model (see Appendix C) meant that no new mappings were required. The index rainfall $RMED_{1d}$ and the 1-day curvature parameter $c_{24} \equiv g \equiv a$ are needed but have already been mapped in Sections 4.2 and 4.3.1 respectively. The parameters h and s of the sub-daily part of the rainfall DDF model are calculated at any point by means of a scheme summarised in Table C.1 of Appendix C.

4.4 Consistency of rainfall depth estimates

4.4.1 Consistency requirements for daily part of DDF model

The daily part of the rainfall DDF model has: $c_D = a + b\ \ell n\ D$ and $s_D = e + f\ \ell n\ D$. To meet the requirement that (for a given return period T) the estimate should increase with duration, it is necessary to have:

$$e + 2\ f\ \ell n\ D + b\ \ell n\ (T\text{-}1)\ >\ 0 \qquad 4.6$$

At about T = 1100 years and D = 1 day, this becomes:

$$e + 7\ b\ >\ 0 \qquad 4.7$$

For a given duration, to have a positive rate of growth with return period T requires:

$$a + b\ \ell n\ D\ >\ 0 \qquad 4.8$$

At about D = 33 days, this becomes:

$$a + 3.5\ b\ >\ 0 \qquad 4.9$$

4.4.2 Consistency for Irish data (1 to 25 days)

Problems were found at long return period (~1000 years) for nearly 7% of cases, where the 1-day depth estimate may be slightly greater than the 2-day estimate. The basic reason is that the curvature parameter *c* is decreasing too sharply. This was corrected by enforcing Condition 4.7. Meeting Condition 4.8 did not present a problem.

4.4.3 Consistency for Northern Ireland data (1 to 25 days)

The parameter values for the Irish data for 1 to 8-day durations had similar quartiles to those for the 1 to 25-day durations (see Table A.1 in Appendix A). Hence, it was decided to use the Northern Ireland data as if it were for periods up to 25 days. This was a convenient though risky assumption.

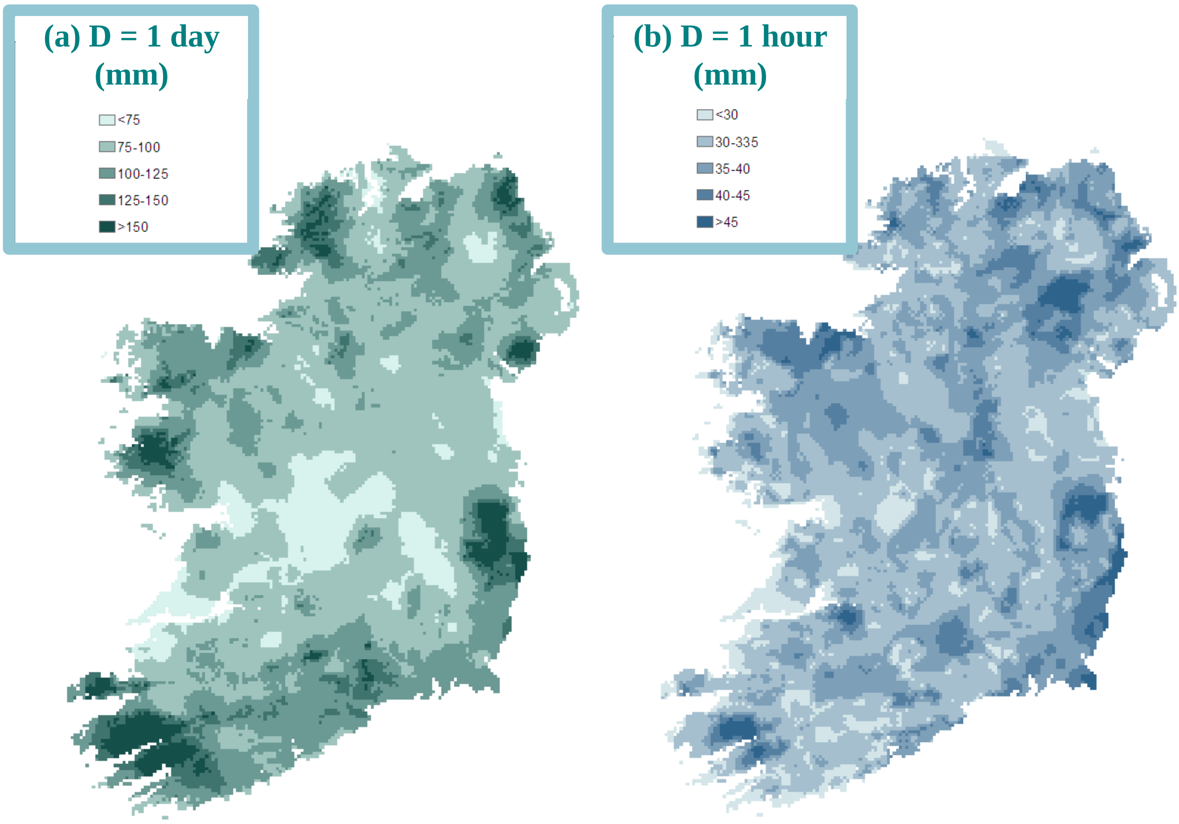
Condition 4.7 had to be imposed in nearly 17% of cases. This adjustment also remedied the failure of 8% of the stations to meet Condition 4.9.

4.4.4 Consistency at sub-daily durations

Because of the detailed manner and special rules used in formulating values of the parameter h, consistency problems did not arise in the sub-daily part of the rainfall DDF model. Close reading of Appendix A of Fitzgerald (2007) is required to fully appreciate the considerable care taken in devising this part of the rainfall DDF model.

4.5 Sample maps

The DDF procedure is fully generalised and can be applied at any site in Ireland. By way of illustration, Map 4.2 shows 100-year 1-day and 100-year 1-hour rainfall depths obtained with the procedure. Estimates in Northern Ireland can be compared with those obtained by Faulkner (1999).



Map 4.2: Sample maps of 100-year rainfall depths

5 Reliability of the estimates of return-period rainfalls

These matters are discussed in more detail in Appendix D, where the tentative conclusions are drawn that:

- The 24-hour to 600-hour estimates may be used with fair confidence for return periods up to about 500 years;
- The estimates for durations of less than 24 hours may be used with fair confidence for return periods up to about 250 years.

The statistical analysis was done on the assumption that data from 1941-2004 are representative of the future rainfall regime. Given concerns about probable changes in precipitation climate in the short to medium term due to global warming, this is by no means assured. How to adjust the estimates is not a question to which there is a good answer at present. Some remarks are nevertheless made in Chapter 6.

Accepting the current consensus on the high likelihood of changes in the precipitation climate, there might appear to be little sense in estimating 500-year rainfall depths. However, rare events occur somewhere in most years, and it is a matter of chance when a very rare event will next affect a specific site. Applying the risk equation (Box 2.2), we note that the 500-year rainfall has a 9.5% chance of being exceeded over a time horizon of 50 years. [**Editorial note**: Fifty years might correspond to the intended design life of a structure or the projected career-span of a flood professional.]

Appendix D provides a method for attaching a standard error to any estimated return-period rainfall from knowledge of the values of the median rainfall and the curvature parameter, plus a rather large assumption about the effective sample size. This statistical measure of spread gives an idea of the probable accuracy of the value. It should be regarded as no more than a rough guide.

While the past is not a secure guide to the future, publications such as Rohan (1986) and Hand *et al.* (2004) provide valuable information about rainfall extremes experienced over the last 100 years or so.

6 Probable effects of climate change on extreme rainfalls

At the time of completion of this research, the most recent IPCC report on regional climate projections (Christensen *et al.*, 2007) stated that over Northern Europe between 1980-1999 and 2080-2099 the median change projected in precipitation is a 15% increase for the months of DJF, 12% for MAM, 2% for JJA and 8% for SON. Relative to the wettest period in 1980-1999, there is a projected 43% increase in wet events in DJF. These projections are highly generalised and based on a relatively coarse model-resolution of ≈200 km.

According to a 2007 assessment from the Community Climate Change Consortium for Ireland (C4I), by mid-century, there may be over Ireland:

- An increase of about 15% in winter rainfall amounts;
- Drier summers with 20% lower precipitation in some areas, most likely the East and South-East;
- A 20% increase in 2-day maximum rainfalls, especially in Northern areas (with smaller increases projected in 1 and 5-day maximum rainfalls);
- More frequent large rainfalls in autumn.

All this would suggest an increase in extreme rainfalls for durations of 24 hours or more, especially in autumn-winter. Drier summers suggest an increase in the frequency of droughts. The breakdown of droughts is sometimes the occasion for heavy short-duration rainfalls.

The general suggestion of most of the scenarios is that safety factors of maybe as much as 20% on rainfall depth might be incorporated as an attempt at a “no regrets policy” in the face of uncertainty.

A purely statistical exercise in Fitzgerald (2005) comes up with safety factors for 1-day rainfalls at Phoenix Park (Dublin) of about 11% for a 20-year return period rainfall, 19% for the 100-year value and 33% for 1000-year rainfall. This analysis was based on 122 years of daily data.

Given the wide variation in predictions from assessment to assessment, and between different models and methods, it is advisable always to seek the latest guidance on the likely effects of climate change on extremes of precipitation. One might then consider whether and how to make an explicit adjustment to the estimates of return-period rainfall presented in the Flood Studies Update.

7 Comparisons with TN40 estimates (Logue, 1975)

7.1 Introduction

Until Fitzgerald (2007), the methods used to compute design rainfalls in Ireland have typically been based on Met Éireann TN40 (Logue, 1975). The methods were those of the FSR – the UK Flood Studies Report (NERC, 1975) – adapted to Irish conditions.

The FSR methodology was to fit a set of growth curves based on the generalised extreme value distribution to series of annual maxima. The method used *two* index rainfalls: the 2-day 09:00-09:00UTC rainfall depth of 5-year return period and the 1-hour rainfall depth of 5-year return period. Growth curves were generalised for use at any site, chiefly structured in terms of duration and average annual rainfall.

The FSU methodology uses the log-logistic distribution as the growth curve and the median of annual maxima (of the relevant duration) as the index rainfall.

7.2 Basis of comparisons

For an average annual rainfall of 1100 mm, tables from TN40 can be used to characterise the rainfall growth curve in terms of the curvature parameter of the log-logistic distribution. This was done by extracting the ratio of the 50-year rainfall depth to the 2-year rainfall depth from Table III of TN40, and inserting the value into the rainfall DDF model of Equation 2.4:

$$R(T,D) = R(2,1)\,D^{s}\,(T-1)^{c}$$

Substituting T= 50, and revealing the M50 and M2 notation used by NERC (1975):

$$\frac{M50}{M2} \equiv \frac{R(50,D)}{R(2,D)} = \frac{R(2,1)\,D^{s}\,(50-1)^{c}}{R(2,1)\,D^{s}\,(2-1)^{c}} = (50-1)^{c} = 49^{c} \qquad 7.1$$

Inversion of this equation forms the basis of the equivalent values of c listed in the TN40 row of Table 7.1.

The comparison was anchored at the FSU end by taking values for the 1-day duration of the parameters of the DDF model corresponding to the near national-average SAAR of 1100 mm. This yielded a 1-day value of c of 0.19. This was used with a rate of change of c with ℓn D of -0.028 for durations longer than one day and of -0.010 for durations shorter than one day.

Table 7.1: TN40/FSU comparison in terms of mean (log-logistic) curvature parameter, c

Study	15 mins	30 mins	1 hour	2 hours	4 hours	6 hours	12 hours	1 day	2 days	4 days	8 days	16 days	25 days
TN40	0.240	0.230	0.220	0.200	0.185	0.180	0.170	0.160	0.155	0.130	0.120	0.110	0.105
FSU	0.235	0.230	0.220	0.215	0.210	0.200	0.195	0.190	0.170	0.150	0.130	0.110	0.100

It should be recalled that – in contrast to the FSR method – durations of 24 hours and one day are exactly equivalent in the final FSU model. Another point of detail is that TN40 employs calendar-month rainfall depths at longer duration. As in NERC (1975), these are taken to be equivalent to 25-day maximum rainfalls.

7.3 Appraisal

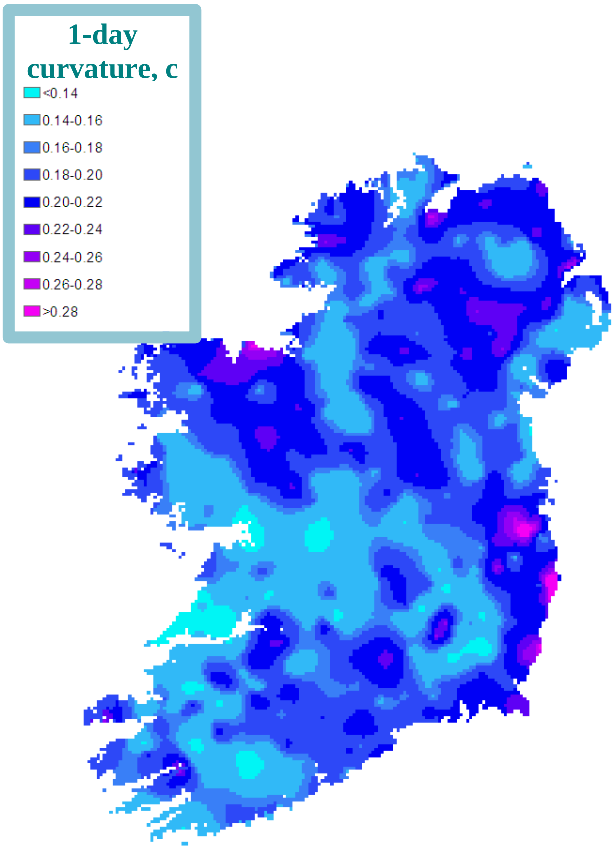
At first sight, agreement between the TN40 and FSU representations of the curvature parameter is impressively strong, with the mean values differing appreciably only in the range 4 hours to 4 days, and centred on the 1-day duration. However, there are significant differences in the *ranges* of the curvature parameter.

For example, in TN40, the 24-hour curvature parameter ranges from 0.10 to 0.19 and the 1-hour curvature parameter ranges from 0.16 to 0.26. The highest values of c are in areas of low SAAR, while the lowest values are in mountainous areas. In the FSU study, the ranges of the curvature parameters are wider. For example, the 1-day curvature parameter varies between 0.11 and 0.30. Map 7.1 confirms that some of the lowest values of the parameter are in lowland areas. This represents a marked difference between the FSU and TN40 models.

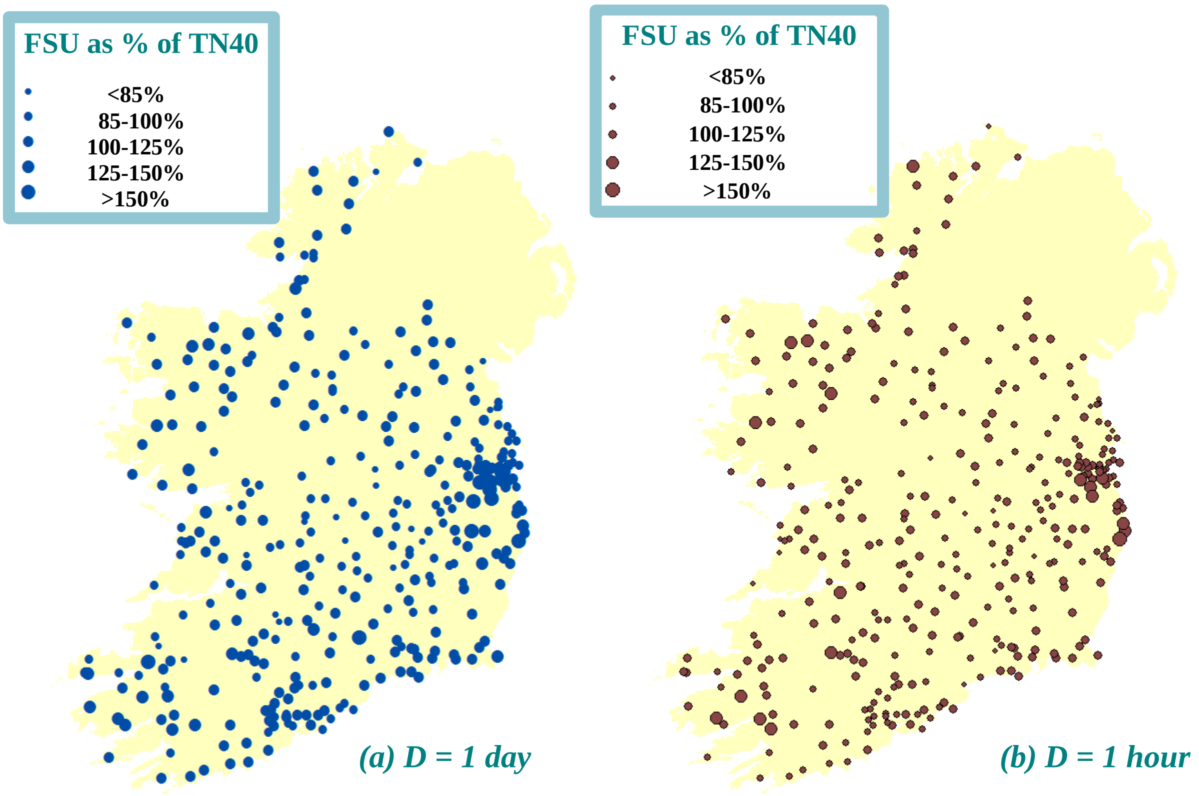
The curvature parameter is particularly influential in the estimation of rarer events. This helps to explain the rather wide variations in 50-year 1-day rainfall depths between the FSU and TN40 models – see Map 7.2a. While there is no strong bias for the FSU method to produce greater 50-year depths than by the TN40 method, a broad spatial pattern is evident in Map 7.2a. Namely, estimates of the 50-year 1-day depth are often higher by FSU (than the TN40 equivalent) towards the coasts, and are sometimes lower in central parts of Ireland.

Spatial patterns in estimates of the 50-year 1-hour depth (Map 7.2b) are less clear-cut. However, it is noticeable that the FSU values tend to be greater than the TN40 values in the Dublin area. This may reflect any of a number of factors. While it might be tempting to attribute the higher 50-year 1-hour depths to an increase in the urban heat island of Dublin in recent decades, it is very much more likely that the feature arises from some difference – between TN40 and FSU – in the gauging network used or in the analysis/modelling method adopted.

[**Editorial note**: The symbol sizes and skewed sub-ranges in the maps do not assist clear interpretations. The symbolisation accords cases where the FSU estimate is higher very much greater visual impact than cases where the TN40 estimate is higher. Applications will no doubt confirm locations for which 50-year depths differ appreciably between the two methods.]



Map 7.1: Values of curvature parameter for 1-day duration (FSU model)



Map 7.2: Comparison of FSU and TN40 estimates of 50-year rainfall depths

8 Miscellaneous matters

8.1 Rainfalls of durations less than 15 minutes

Dines recorders provided the bulk of the data for rainfalls shorter than 24 hours. These recording raingauges are not accurate below 15 minutes (Logue, 1975). Nonetheless, estimates of the daily maximum rates over a period of about 5 minutes have been extracted at Irish synoptic stations over periods of 30 to 50 years. Maximum 5-minute values at the stations range between 8 and 12 mm.

The sub-daily rainfall DDF model properly applies to durations between 24 hours and 15 minutes. Its extrapolation to shorter durations presents some problems. This is largely because the curvature parameter – controlling the steepness of the rainfall growth curve – increases as duration decreases. There is the possibility that extrapolated 5 or 10-minute values will be unrealistic.

The recommended approach is instead to employ the hydrological concept of a rainfall profile of given duration, e.g. Chapter II.6 of NERC (1975). Under this approach, the 5 or 10-minute fall is regarded as a sub-period of the 15-minute rainfall event. The 15-minute depth estimate is taken to be correct, and the required 5-minute or 10-minute estimate is factored from the 15-minute estimate by applying a standard rainfall profile.

Chapter 4 of Faulkner (1999) provides a formula for the 75% winter profile developed by NERC (1975). While use of this profile may appear to introduce bias – because it is explicitly defined to be peakier than 75% of winter storms – this is possibly off-set by under-measurement of maximum rainfall depths over very short durations.

8.2 Catchment rainfalls

8.2.1 Requirements

In certain "design event" methods of flood estimation, the hydrological requirement is to estimate the T-year catchment rainfall rather than the T-year point rainfall. Analogously, when investigating a particular extreme event, it can be valuable to estimate the rarity of the catchment rainfall depth that has led to flooding.

Earlier (non-digital) methods evaluated catchment rainfall frequency in three steps:

- Selection of a typical point within the catchment
- Evaluation of the rainfall frequency relationship for this typical point;
- Application of an areal reduction factor (ARF) to estimate the catchment rainfall frequency.

In the FSU (digital) procedure, the first two steps are merged.

8.2.2 Rainfall frequency relationship for a typical point in the catchment

Using the FSU Web Portal, the T-year rainfall for a typical point within the catchment is obtained by direct averaging of the T-year rainfall evaluated at every 2-km grid point within the catchment boundary. The procedure is summarised in the right-hand branch of the flowchart in Figure 8.1.

If the subject catchment is smaller than 5km^2, the 2-km grid point closest to the catchment centroid is taken as the typical point within the catchment, and no averaging is carried out. This case is summarised in the left-hand branch of the flowchart. Small catchments are sometimes associated with steep terrain. The user is therefore warned to check that the grid-point altitude is typical of the catchment, and the software tool allows the user to select a different 2-km grid point.

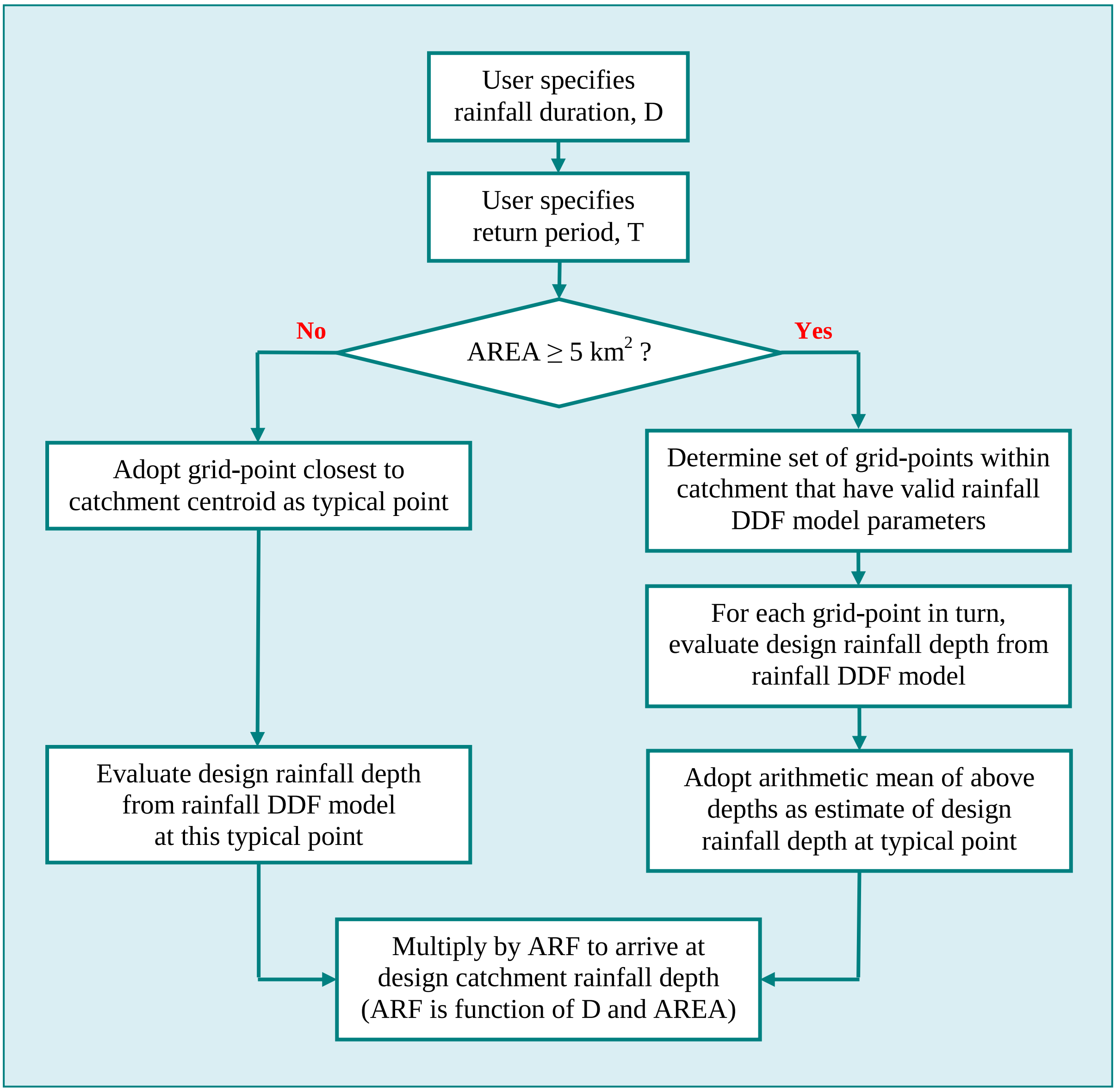


Figure 8.1: Flowchart of procedure for estimating catchment rainfall depth, R(T,D)

8.2.3 Application of areal reduction factor (ARF)

In all cases, it is necessary to apply an areal reduction factor to reduce the T-year rainfall for a typical point to obtain the required estimate of the T-year catchment rainfall. Chapter 3 of Faulkner (1999) succinctly summarises the need to apply an areal reduction factor (ARF):

> *Because rainfall is rarely uniform, particularly in extreme storms, the T-year rainfall at a point is bound to be larger than the T-year rainfall over an area. Viewed another way, the atmosphere has to work much harder to exceed a given rainfall depth over a 100 km^2 catchment than it does to exceed the same depth at one raingauge.*

Faulkner helpfully republishes a formula developed by Keers and Wescott (1977). This defines ARF as a function of catchment size (AREA km^2) and rainfall duration D, which Keers and Wescott take in hours rather than days.

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Volume II was edited by Duncan Reed of DWRconsult, who added Section 8.2.

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Appendices

Appendix A Development of the DDF relationship

A1 Motivation behind use of log-logistic distribution

The log-logistic distribution is put forward for rainfall frequency analysis by Fitzgerald (2005), who notes properties that make it suited to use as a growth curve. The parameterisation differs from the Generalised Logistic distribution considered by (e.g.) Hosking and Wallis (1997) and Robson (1999b), and referred to in Volume II of the Flood Studies Update. Some elaboration is therefore appropriate.

The log-logistic is defined by the probability density function (PDF):

$$f(x) = \frac{1}{ac} \frac{[(x-g)/a]^{\frac{1}{c}-1}}{\left(1+[(x-g)/a]^{\frac{1}{c}}\right)^2} \qquad \text{A.1}$$

The cumulative distribution function (CDF) of the log-logistic is then:

$$F(x) = \frac{[(x-g)/a]^{1/c}}{1+[(x-g)/a]^{1/c}} \qquad \text{A.2}$$

[**Editorial note**: The formulations used here are based on Fitzgerald (2005) but with 1/c replacing b. Those comparing sources should note that typographical error led to omission of the innermost set of brackets in the denominator of the PDF in Fitzgerald (2005). Fitzgerald (2007) refers to c as the shape parameter. It is termed the curvature parameter in the present volume to distinguish it more clearly from the scale parameter s also used.]

It can be seen from Equation A.2 that the median (i.e. F = 0.5) of the log-logistic occurs when:

$$\frac{x-g}{a} = 1$$

This makes the log-logistic distribution a particularly convenient choice for the growth curve when the median of annual maxima is chosen as the index variable. [**Editorial note**: The median of the annual maximum series is also used as the index variable in Volume II, where it is denoted by QMED.]

Replacing x in Equation A.2 by R(T, D) – i.e. the T-year rainfall depth of duration D – and rearranging, we have:

$$\frac{R(T,D)-g}{a} = \left(\frac{F}{1-F}\right)^c \qquad \text{A.3}$$

Noting from Equation 2.2 that return period T is defined as 1/(1-F), we get:

$$\frac{R(T,D)-g}{a} = (T-1)^c \qquad \text{A.4}$$

Inserting T = 2, it is confirmed that the median of the annual maxima – i.e. the index rainfall R(2, D) – corresponds to the sum of the location parameter (g) and the scale parameter (a) of the log-logistic:

$$R(2, D) = g + a \qquad \text{A.5}$$

This adds to the appeal of using the log-logistic distribution as the basis of the rainfall growth curve, when adopting R(2, D) as the index rainfall.

[**Editorial note**: In the present study, the above 3-parameter form of the log-logistic is used when modelling AM rainfalls of an individual duration. However, when used as an element of the overall rainfall DDF model, the location parameter (g) is set to zero so that the scale parameter a is just the median AM rainfall of the relevant duration, i.e. R(2, D).]

A2 AM data series

Two sets of annual maximum (AM) data were available:

- 09:00-09:00UTC rainfall accumulations for 11 durations between one and 25 days, all being fixed-duration rainfalls;
- Short-duration series for nine durations ranging from 15 minutes to 24 hours, all being sliding duration rainfalls.

There are some fundamental differences between the two sets, which are distinguished here by the qualifiers *daily data* and *sub-daily data*.

A2.1 AM series for daily data

Because of the work on data infill (see Section 3.1 and Appendix B), the AM series based on daily rainfall data are essentially complete. They are based on observations in fixed periods of 09:00-09:00UTC. These are known as *fixed-duration rainfalls*.

A2.2 AM series for sub-daily data

A feature of the AM series based on sub-daily data is that they have been abstracted from (effectively) continuous rainfall records. Thus, the 1-hour AM rainfall represents the largest rainfall recorded in any 60-minute period, i.e. it is not constrained to a period ending on the clock hour. These are therefore known as *sliding-duration rainfalls*.

A further feature is that the AM series for sub-daily data have missing values. These arise in years when the annual maximum rainfall is below the threshold used in data abstraction. The AM series is said to be *left-censored* (i.e. censored below). In essence, the magnitudes of annual maxima that fall short of the threshold are unknown.

The censored nature of the AM series will usually require special treatment, as is typically the case in the analysis of historical flood data when the dates of exceedances above a high threshold are known but their magnitudes are not. Intricate methods of parameter estimation for left-censored samples were therefore developed for the log-logistic distribution and are reported by Fitzgerald (2007). However, because our interest is in large values, the loss of information about small annual maxima is relatively unimportant. Moreover, it transpired that the censoring did not in fact compromise the method finally adopted for deriving the rainfall DDF model (see Section A3.4).

A3 Exploratory data analysis

A3.1 Results for individual durations

The 3-parameter log-logistic distribution was fitted to annual maxima for individual durations and sites (i) by maximum likelihood (ML) methods and (ii) by the method of probability-weighted moments. Both approaches are outlined in Fitzgerald (2005). This exploratory analysis revealed that – for daily and sub-daily datasets alike – estimates of the log-logistic parameters exhibited too irregular a variation with duration to be suitable for directly fitting a DDF relation to individual stations.

A3.2 Sub-daily dataset

Judged from the average over the 39 stations in the sub-daily dataset, the mean of the log-logistic curvature parameter c increased (albeit somewhat unsteadily) as the duration reduced below 24 hours, reaching a maximum at one hour. Values of c at 30 and 15 minutes were slightly lower.

The scale parameter s expressed as a fraction of the median showed little pattern with duration beyond a tendency for the lowest values to occur in the 1 to 4-hour range. There was sometimes unhelpful interaction between the c and s parameters, with undesirably high values of the curvature parameter ($c \geq 0.4$) compensating for low values of the scale parameter s.

A3.3 Daily dataset

For the 474 daily stations (based on 09:00-09:00UTC data), the mean value of the curvature parameter c decreased slowly as the duration increased from one or two days to 25 days. The scale parameter s expressed as a fraction of the median showed its highest values at long duration, with the lowest values in the 1 to 3-day range. Overall, though, the variation with duration was erratic.

A3.4 Exploratory form of DDF model

An early attempt to stabilise the estimates put the log-logistic distribution in the growth-curve form:

$$\frac{R(T,D)}{R(2,D)} = 1 + K_D\left[(T-1)^{c_D} - 1\right] \qquad \text{A.6}$$

where

$$K_D = \frac{a(D)}{R(2,D)} \qquad \text{A.7}$$

can be thought of as a frequency factor.

The parameters c_D and K_D were expressed as functions of duration, for which various forms were tried. The most useful proved to be:

$c_D = a + b\,\ell n\,D$ A.8 and $K_D = e + f\,D$ A.9

Fitting the exploratory model

Annual maxima were extracted from the full datasets, and standardised by division by the sample median to provide values for the left-hand side of Equation A.6. Only standardised values greater than or equal to 1.0 were used in the modelling. [**Editorial note**: Neglect of the lower half of the AM data is reasonable given that the rainfall DDF model is chiefly intended for use for $T \geq 2$.]

Helpfully, all series of maxima greater than or equal to the median were complete. There was therefore no need to employ methods of parameter estimation for censored samples. As suggested by Koutsoyiannis *et al.* (1998), the model parameters were estimated simultaneously.

Results

In general there was good agreement between the PWM/ML solutions for individual durations and those obtained using the portmanteau solution provided by Equations A.8 and A.9. However, as with the PWM/ML solutions, the values of c_D and K_D fluctuated markedly between neighbouring stations: making it difficult to detect any pattern.

A4 Final form of DDF model for durations of one day and longer

A4.1 Structure

The final form of the rainfall DDF model takes the log-linear form:

$$\frac{R(T,D)}{R(2,1)} = D^{s}(T-1)^{c} \qquad \text{A.10}$$

Note that D is measured in days, regardless of whether multi-day or sub-daily durations are being considered. The 2-year 1-day rainfall – i.e. the median of the AM 1-day rainfalls – is adopted as the index rainfall.

Noting that: $\frac{R(T,1)}{R(2,1)} = (T-1)^{c}$ and $\frac{R(T,D)}{R(2,1)} = D^{s}(T-1)^{c}$, we have by division:

$$\frac{R(T,D)}{R(T,1)} = D^{s} \qquad \text{A.11}$$

Thus, the variation across durations is a simple power-law.

Similarly, noting that: $\frac{R(T,D)}{R(2,1)} = D^{s}(T-1)^{c}$ and $\frac{R(2,D)}{R(2,1)} = D^{s}$, we have by division:

$$\frac{R(T,D)}{R(2,D)} = (T-1)^{c} \qquad \text{A.12}$$

Comparison with Equation A.4 confirms that the growth curve defining the variation with return period is of log-logistic distribution form, with a location parameter of zero and a scale parameter of R(2, D).

A4.2 Parameters for 1 to 25-day model

For the curvature parameter c, exploratory work (see Section A3.3) had indicated a slow variation with duration. This was assumed to be of the form:

$$c_D = a + b \ \ell n \ D \qquad \text{A.13}$$

Assuming s is constant in Equation A.10 in conjunction with Equations A.12 and A.13 gave good results. However, adopting a comparable sub-model for the exponent s:

$$s_D = e + f \ \ell n \ D \qquad \text{A.14}$$

led to a valuable improvement by reducing the range of the residuals.

The four parameters a, b, e and f were estimated simultaneously using the R routine *lm*, which denotes linear model. The results were excellent, with a coefficient of determination (r^2) in excess of 0.99.

In addition, the level of agreement between the log-logistic PWM/ML quantile estimates and those derived from Equations A.10, A.13 and A.14 was tested for return periods up to 250 years and found to be good. All four parameters were highly significant in the sense of being much larger than their standard errors.

A4.3 Use of daily data for Northern Ireland

Annual maximum 1, 2, 4 and 8-day rainfall depths were made available for daily gauges in Northern Ireland (NI) from CEH research (e.g. Faulkner, 1999). Those 103 stations with more than 20 years of record were used.

Table A.1 compares the NI 1-day c and s values with those derived for the Met Éireann daily stations. Two versions of the Irish modelling are shown: one with the full set of 11 durations (1, 2, 3, 4, 6, 8, 10, 12, 16, 20 and 25 days) and one using the six durations spanning the 1 to 8-day range (1, 2, 3, 4, 6 and 8 days).

It is a mild surprise that the median values for Northern Ireland are higher than for Ireland. However, the standard deviations of c_1 and s_1 are roughly 0.04 and 0.07 respectively. The cross-border differences are therefore not hugely significant, given that a different array of rainfall durations and different periods of record has been used.

It is reassuring that the parameter estimates based on six durations are very similar to those based on 11 durations. This lends some justification to accepting the NI estimates based on four durations within the overall modelling. However, estimates of return-period rainfalls longer than about 10 or 12 days for locations in Northern Ireland should be treated with caution.

Table A.1: Summary values of the 1-day parameters of the DDF model

Model version →	**Northern Ireland (four durations)**		**Ireland (six durations)**		**Ireland (11 durations)**	
Durations used in model fitting →	**1, 2, 4 and 8 days**		**1, 2, 3, 4, 6 and 8 days**		**1, 2, 3, 4, 6, 8, 10, 12, 16, 20 and 25 days**	
Parameter →	c_1	s_1	c_1	s_1	c_1	s_1
Minimum	0.12	0.25	0.08	0.17	0.08	0. 18
Q1	0.17	0.36	0.15	0.33	0.16	0.31
Median	0.20	0.41	0.18	0.37	0.18	0.35
Q3	0.23	0.45	0.20	0.41	0.20	0.38
Maximum	0.32	0.55	0.34	0.56	0.35	0.51

A4.4 Effects on parameter estimates of converting to sliding durations

Table A.1 is based on the analysis of fixed-duration 09:00-09:00UTC data. The DDF model was refitted after the various AM rainfall depths had been converted to sliding durations by multiplying by the appropriate adjustment factors (i.e. Table 3.4). The curvature parameter c of the growth curve was unchanged but the duration exponent s decreased by 0.09.

A4.5 Implementation in outline

As described in Section 4.3, station values of the four parameters required by Equations A.13 and A.14 – and also station values of the index rainfall R(2, 1) – were extrapolated to a 2-km grid. At grid points, Equation A.10 could then be used to derive the return-period rainfall depths for any duration between one and 25 days.

Note that all model outputs are for sliding durations. If fixed-duration rainfalls are required (e.g. for assessing the rarity of a rainfall depth observed by daily-read gauge), it is necessary to divide the sliding-duration rainfall by the appropriate adjustment factor (see Table 3.4).

Appendix B Data infill methods

B1 Where SAAR comparable

For most stations requiring data infill, estimations were made by using up to six neighbouring stations having similar (within about 10%) standard-period average annual rainfall (SAAR). Potential donor stations were ranked in order of preference according to proximity to the subject station. The first such station having a complete record for the period requiring estimation was generally used. This had the perceived advantage of using a daily total actually recorded in the general area rather than a weighted average of totals.

B2 Where SAAR differs considerably

A more complicated approach was required when the SAAR of the subject station (requiring infill) differed considerably from each of its nearest neighbours. In such cases, three donor stations with complete daily records were chosen from the general neighbourhood.

B2.1 Entire month missing

Where one or more daily observations are missing (as opposed to accumulated), it is commonplace for the daily data to be declared missing for the entire month and to be omitted from the archive. A two-step approach was used to fill such gaps.

The long-term ratio of monthly rainfall at the subject station to that at the donor station was examined for each of the three donor stations. The missing monthly total at the subject station was estimated *pro rata* using each of the donors in turn. The monthly total adopted was taken as a distance-weighted mean of the three donated monthly values. [**Editorial note**: It is believed that the weight accorded to each donor station was in proportion to the reciprocal of its distance from the site requiring infill.] The infilled monthly depth was then disaggregated to daily values by reference to the profile of daily rainfalls at the nearest neighbour.

B2.2 Monthly total available but no daily values

Months with a total but no daily values were treated by forming the weighted mean monthly depth of the three neighbours and comparing this to the monthly total for the subject station. Agreement between the two estimates was usually good, and their mean was adopted as an adjusted monthly value for the subject station. This was then disaggregated to daily values by reference to the profile of daily rainfalls at the nearest neighbour.

B2.3 Monthly total available, most daily values available

Isolated missing days with missing or accumulated totals were treated by multiplying the daily values at the nearest available neighbour by the ratio of the monthly depths at the subject and donor sites.

B3 Outcome

Annual maxima extracted from the original and treated data confirmed that differences were usually small. This is thought to reflect the high quality of the chosen stations.

Appendix C Final form of the 24-hour to 15-minute model

C1 Summary

The analysis and modelling of sub-daily data were highly intricate. The specialist researcher is referred to Appendix A of Fitzgerald (2007). A few points of detail are summarised below.

One matter to resolve was the marrying of median AM 1-day rainfalls (from the analysis of daily data) with the median of AM 24-hour sliding rainfalls. In fact, the sub-daily database available provided absolute maximum values of sub-daily duration rainfalls (i.e. sliding to a time resolution appreciably less than 15 minutes, rather than merely to clock-hour resolution).

As explained in Section 3.2, the maximum rainfalls at sub-daily durations were available as a series of threshold exceedances, i.e. as a peaks-over-threshold (POT) series rather than as an AM series. However, only the upper half of the AM series is required to define the median, as is amply demonstrated by Robson (1999a). It transpired that the POT format of the sub-daily maximum rainfalls proved adequate.

From exploratory analysis, it was found reasonable to take the sub-daily sub-model in the same broad form as the daily sub-model (i.e. Equation A.10):

$$\frac{R(T,D)}{R(2,1)} = D^{s}(T-1)^{c} \qquad \text{C.1}$$

except that:

- The scale parameter s is taken to be constant, rather than varying with D;
- The curvature parameter c – whilst still varying with D – is constrained to match the value of c in the calibrated daily sub-model, where the sub-models “meet” at a duration of 1 day (i.e. 24 hours); as discussed in Box 2.1, this value is specified by parameter a in the daily part of the model, parameter g in the sub-daily part of the model, and variously referred to as c_1 or c_{24}; all these terms are identical ($c_1 \equiv a \equiv g \equiv c_{24}$).

For intricate reasons explained by Fitzgerald (2007), the median sliding 24-hour rainfall $RMED_{1d}$ – derived from the median 1-day rainfall by applying the Section 3.4 conversion factor from fixed-duration to sliding-duration depths – was used in place of the absolute maximum 24-hour depths. The reader is also referred to that report for further details of the form, calibration and testing of the sub-daily sub-model. Only the final scheme is noted below.

C2 Final scheme

The sub-daily part of the DDF model is given by:

$$R(T,D) = RMED_{1d}\, D^{s}\, (T-1)^{c_{24}+h\ell nD} \qquad \text{C.2}$$

where the median sliding 24-hour rainfall $RMED_{1d}$ has replaced R(2,1).

In choosing the final values of the sub-daily parameters s and h, a partitioned approach was used according to the value of $RMED_{1d}$. The final scheme is detailed in Table C.1. Note that the parameters c_{24} and g are known from the daily part of the DDF model.

C3 Implementation

Since grids of $RMED_{1d}$ and a (≡ c24) were already available from Sections 4.2 and 4.3.1 respectively, grids of return period rainfalls for any duration could be generated using the Table C.1 scheme to supply values of s and h.

Table C.1: Final scheme used to set parameters of sub-daily part of DDF model

Range of $RMED_{1d}$	s parameter in Equation C.2	h parameter in Equation C.2
$RMED_{1d} \geq 60$ mm	0.48	-0.01 if $c_{24} < 0.15$
		0 otherwise
47 mm $\leq RMED_{1d} <$ 60 mm	0.43	-0.015 if $c_{24} < 0.16$
		0 otherwise
35 mm $\leq RMED_{1d} <$ 47 mm	0.375	-0.01 if $c_{24} \geq 0.25$
		-0.015 if $0.16 < c_{24} < 0.25$
		-0.023 if $c_{24} \leq 0.16$
$RMED_{1d} <$ 35 mm	0.33	-0.015 if $c_{24} \geq 0.25$
		-0.023 if $0.16 < c_{24} < 0.25$
		-0.030 if $c_{24} \leq 0.16$

The mappings produced in this way were mostly satisfactory but, especially at durations of 1 hour or less, there were a few anomalous spots indicating sudden jumps in the rainfall values. Both the number and extent of these anomalies were small but to deal with them further smoothing was used.

The smoothing scheme applied at each grid point was to add in the values of the four nearest neighbours, weight all five equally, and take the mean as the grid-point value. While this may seem like over-smoothing, especially at the longer durations, the only visible effect on the mappings was to remove the anomalous spots. In view of this, the matter of the degree of smoothing was not pursued further.

Appendix D Some checks and confidence intervals

D1 Introduction

Here the gridded estimates of return-period rainfall close to gauged sites are checked against estimates provided by fitting the log-logistic distribution directly to the rainfall data. A crude method of estimating the standard error of quantile estimates for the gridded data is tested.

D2 Durations of one to 25 days

All rainfall data and estimates in this section are for fixed-duration daily data, i.e. for periods ending at 09:00UTC.

D2.1 Four parameters

The daily part of the DDF model has four parameter: a, b, e and f. The parameter a is identical to the 24-hour (or 1-day) curvature parameter, c_{24}. It is quite feasible to write down expressions for the standard errors of quantiles of the 4-parameter model. The correlation matrix is remarkably constant over the stations. However, the values were obtained using the index rainfall R(2, 1), i.e. the median 1-day rainfall, as if it were a known constant in the DDF model:

$$\frac{R(T,D)}{R(2,1)} = D^{e+f\,\ell nD}(T-1)^{c_{24}+b\,\ell nD} \qquad \text{D.1}$$

Since the sample median was used, estimates of the standard error of R(2, D) should allow for the standard error of R(2, 1) and the covariance of R(2, 1) with each of the above four parameters e.g. Cov[R(2, 1), c_{24}] = -0.0381 while Cov[R(2, 1), e] = 0.0513 over the 577 stations. What this means for a single location is not quite clear but it is assumed that the sign of the inter-station covariance applies. The covariances are small but so too are the terms based on Var[c_{24}] and Var[e]. Thus, strings of small terms are obtained.

D2.2 An assumption

Having low confidence in this approach, a crude but direct method was adopted. The basic assumption made is that – since the model produces quantile estimates similar to those obtained by fitting the log-logistic distribution to individual durations – the model standard error for a given duration will be similar to that derived from log-logistic theory for the individual duration.

D2.3 Some theory

On making this assumption, results of bootstrapping exercises and those of maximum likelihood (ML) theory (Fitzgerald, 2005) can be used. For a given duration, Equation D.1 is rewritten:

$$R(T,D) = R(2,D)\,(T-1)^{c_D} \qquad \text{D.2}$$

To scale down R(2,D) – which in the final DDF model is for sliding durations – to its fixed-duration value, the adjustment factors of Table 3.4 are again used.

Maximum likelihood (ML) theory gives:

$$\frac{\mathrm{Var}(R(T,D))}{R(T,D)^2} = \frac{\mathrm{Var}(R(2,D))}{R(2,D)^2} + \mathrm{Var}(c_D)(\ln(T-1))^2 \qquad \text{D.3}$$

and

$$\mathrm{Cov}(R(T,D),c_D) = 0 \qquad \text{D.4}$$

The negative value of Cov[R(2, 1), c_{24}] noted in Section D2.1 suggests that Cov[R(T, D), c_D] may well be negative, making the assumption of a zero value acceptable in that it increases the variance.

Further, for any duration D:

$$\mathrm{Var}(c_D) = \frac{9c_D^{\ 2}}{(\pi^2+3)n} \qquad \text{D.5}$$

and

$$\frac{\mathrm{Var}\left(R(2,D)\right)}{R(2,D)^2} = \frac{3c_D^{\ 2}}{n} \qquad \text{D.6}$$

Plunging on, it is now assumed that despite the very different methods of arrival at the values of R(2, D) and c_D, the above ML formulae apply also to the gridded rainfall estimates (see Chapter 4) with n, the sample size, set to 41 years. This is the average record-length across the 577 stations in the daily dataset.

D2.4 Standard errors in three cases

Standard errors can now be examined, and are denoted by se(ML) to indicate their basis in maximum likelihood (ML) theory. Three cases are considered, in which 2p, 3p and 4p denote 2-parameter, 3-parameter and 4-parameter models:

2p/3p The L-moment solution for the series of annual maxima for individual durations, with the location parameter regarded as fixed but initially unknown (Fitzgerald, 2005);

4p-data The solution based on using those annual maxima greater than or equal to the sample median for the 11 durations (1, 2, 3, 4, 6, 8, 10, 12, 16, 20 and 25 days) to simultaneously estimate the four parameters of the DDF model;

4p-grid The solution based on interpolated grid-values of the four parameters of the DDF model, assuming equivalence to estimates from 41 years of record.

D3 Illustrative application to daily rainfall data for Phoenix Park

For Phoenix Park, 122 years of daily data were available. This gave the opportunity to compare estimates from a very long record (1881-2002) with the 1941-2004 period on which the FSU rainfall DDF model has been based.

D3.1 Parameters of log-logistic for the three cases

From the annual maximum series and from the grids of the parameters we get the parameters shown in Table D.1. Two variants of the at-site analysis are shown: one based on 1941-2004, and one based on 1881-2002.

Table D.1: Parameter values for Phoenix Park 1-day AM rainfall

Case	Curvature parameter	Scale parameter	Location parameter
2p/3p (based on 1881-2002)	0.274	22.2	12.0
2p/3p (based on 1941-2004)	0.300	20.0	14.1
4p-data (i.e. at-site analysis)	0.224	34.6	0.0
4p-grid (i.e. final DDF model)	0.239	33.0	0.0

Recall that, for the 2p/3p case, the median is given by the sum of the location and scale parameters (see Section A1). In the 4p-data and 4p-grid cases, the location parameter is zero and the scale parameter is the median.

D3.2 Quantile estimates and standard errors by the three approaches

Table D.2 gives the return-period rainfalls and their standard errors for 1-day (09:00-09:00UTC) AM rainfall data for the period 1941-2004 at Phoenix Park. The three cases are shown in the three pairs of columns.

Table D.2: Phoenix Park 1-day quantile estimates and their standard errors

Based on 1941-2004	Directly fitted model		4-parameter DDF model		4-parameter DDF model	
	At site		At site		Gridded	
Return period	Rainfall depth	se(ML)	Rainfall depth	se(ML)	Rainfall depth	se(ML)
years	mm	mm	mm	mm	mm	mm
2	34.1	1.3	34.6	1.7	33.0	2.1
5	44.3	2.4	47.0	2.8	45.9	3.5
10	52.8	3.7	53.6	4.0	55.7	5.2
20	62.5	5.5	66.9	5.6	66.6	7.4
50	78.4	8.9	82.8	8.6	83.5	11.4
100	93.5	12.5	96.9	11.4	98.7	15.4
250	118.8	19.3	119.1	16.4	123.0	22.5
500	143.1	26.4	139.2	21.4	145.2	29.5
1000	172.9	35.8	162.7	27.5	171.4	38.4

Agreement between the three sets of estimates of the return-period rainfalls is excellent. For the directly fitted (2p/3p) case, and for the 4-parameter DDF model applied at-site, the number of years of record is taken to be 64 (i.e. 1941-2004). For the gridded DDF model case, the number of years of record is taken to be the network average of 41 years. Many of the stations neighbouring the Phoenix Park site are in fact full-period across 1941-2004.

Thus, the assumption of n = 41 is likely a bit low. Nonetheless the estimate of the standard error by interpolation from the gridded parameter values appears reasonable.

D3.3 Testing the quantile estimates and their standard errors

To get an idea of how realistic the above quantiles and standard errors are, Table D.3 compares the ML estimates with bootstrap estimates. Note that, in this comparison, all the quantile estimates and standard errors are based on 122 years of data (1881-2002).

Table D.3: Bootstrap corroboration of ML standard errors

[Phoenix Park, direct analysis of AM 1-day data from 1881-2002]

Return period, years	1-day (09:00-09:00UTC) T-year rainfall depth (mm)	se(bootstrap)	se(ML)
2	34.3	1.0	1.0
5	44.4	1.7	1.7
10	52.4	2.5	2.5
20	61.4	3.6	3.6
50	75.8	5.7	5.7
100	89.2	8.2	8.2
250	111.3	13.0	12.1
500	132.1	18.2	16.3
1000	157.2	25.1	21.7

The agreement between the bootstrap and ML estimates of the standard error is good. The indications from Table D.3 are that even the 500 and 1000-year rainfalls in Table D.2 derived from the gridded values of the parameters are reasonable, "even if it does take some acclimatisation to think of 171 ± 38 mm as being consistent with 157 ± 25 mm" (Fitzgerald, 2007).

The utility of the standard error is that it gives some notion of the reliability of the gridded values in Table D.2. Making the usual normal-distribution assumption, you might regard the 1000-year AM 1-day rainfall as being between 133 mm and 209 mm in 68% of cases.

D3.4 Conclusions for durations of 1 to 25 days

Fitzgerald (2007) also considers a longer duration, obtaining an estimate for the 500-year 25-day AM rainfall of 340 ± 47 mm. Making the usual normal-distribution assumption, you might hopefully regard the 500-year AM 25-day rainfall as being between 293 and 387 mm in 68% of cases.

Given the broad assumptions made in deriving these standard errors, it would hardly be wise to quote (e.g.) 95% confidence intervals based on them.

Taken as what they are – i.e. annual exceedance probabilities based on the assumption that the annual series 1941-2004 adequately represent the future – it is judged that the rainfall DDF model estimates may be used with reasonable confidence for return periods of up to about 500 years.

D4 Confidence of estimates for durations shorter than one day

The same broad ideas can be applied to examine the reliability of estimates from the sub-daily part of the DDF model. Fitzgerald (2007) gives some results for 1-hour rainfall quantile estimates at three locations: Claremorris, Mullingar and Waterford (Tycor). Some use is made of estimates based on the method of partial probability-weighted moments (Wang, 1990). This is a development of probability-weighted moments designed for use with censored samples.

Estimates at Mullingar were particularly challenging, with the parameters h and s have an unusually strong effect on the curvature and scale parameters for 1-hour AM rainfalls.

Given the much lower density of the underlying raingauge network (39 locations in Ireland as opposed to 474 for daily data) it is inevitable that standard errors are proportionately much larger than for rainfall estimates at one day and longer durations. Fitzgerald (2007) considers them tentative, but nevertheless concludes that estimates of sub-daily rainfalls from the gridded DDF model may be used with reasonable confidence up to a return period of about 250 years.

Appendix E Depth-duration-frequency model outputs

E1 Introduction

Several computer programs have been written to produce output from the DDF model, all of the outputs being for sliding durations. These allow for calculation of return-period rainfall depths for specified durations and frequencies at 2-km grid points throughout Ireland.

As certain applications will be for locations other than grid points, a program was written to make the calculation for any location. The location is defined by Easting and Northing coordinates in metres.

A program has also been provided for calculation of rarity estimates at any point location. The user supplies the rainfall depth and duration, in addition to the Easting and Northing.

The programs interpolate the model parameters to the subject location (see Section E3) and then apply the rainfall DDF model. The programs were written with the R programming language and require that grids of the 2-year 1-day rainfall depth and of the DDF model parameters be installed in the R environment. Details of the R language can be found at: http://www.r-project.org/.

E2 Programs supplied

E2.1 Rainfall depth estimates

Three programs have been supplied for rainfall depth estimation:

- For a specified duration and return period, *Grow1.R* produces rainfall depth estimates for every 2-km grid point;
- For a specified duration, *Grow2.R* produces rainfall depth estimates for a predefined range of return periods for every grid point;
- For a specified grid point, *Point1.R* provides rainfall depth estimates for a predefined range of durations and return periods.

Sample output from *Point1.R* is shown in Figure E.1. Illustrative rainfall frequency curves are shown below for rather a wet location in Co Mayo: sub-daily durations in Figure E.2 and for longer durations in Figure E.3.

E2.2 Rainfall rarity estimates

Program *Point2.R* has been supplied for rainfall rarity estimation. If the rainfall depth and duration are given for a specific location, *Point2.R* produces an estimate of the rainfall return period. This program can also be used for rainfall depth estimation if the return period and duration are given.

The R-language version of this program requires installation of the geostatistical package *geoR* (Ribeiro and Diggle, 2001; Diggle and Ribeiro, 2007).

```
      LOCATION
Easting  84000  Northing 266000
SLIDING DURATIONS 1 day to 25 days
                      Return Period Rainfalls(mm)
       2yr   5yr  10yr  20yr  30yr  50yr 100yr 150yr 250yr 500yr
1d     80.2  99.2 112.3 125.8 134.2 145.4 162.0 172.4 186.5 207.4
1.5d   91.5 111.7 125.5 139.7 148.4 160.1 177.1 187.8 202.2 223.4
2d    101.9 123.2 137.7 152.5 161.6 173.7 191.3 202.3 217.1 238.8
3d    120.8 144.2 160.0 176.1 185.9 198.8 217.5 229.2 244.7 267.5
4d    138.3 163.6 180.5 197.7 208.1 221.8 241.6 253.9 270.2 294.0
5d    154.7 181.8 199.8 218.0 229.0 243.4 264.1 277.0 294.1 318.9
6d    170.5 199.2 218.2 237.3 248.8 263.9 285.6 299.0 316.8 342.5
8d    200.6 232.3 253.1 273.9 286.5 302.8 326.2 340.6 359.6 387.0
10d   229.4 263.8 286.3 308.6 322.0 339.5 364.4 379.7 399.9 428.9
12d   257.3 294.2 318.2 342.0 356.2 374.7 401.0 417.2 438.4 468.8
16d   311.3 352.7 379.4 405.8 421.6 442.0 470.9 488.5 511.6 544.7
20d   363.8 409.3 438.5 467.2 484.3 506.4 537.5 556.5 581.4 616.7
25d   428.0 478.2 510.2 541.6 560.2 584.2 617.9 638.5 665.2 703.2
SLIDING DURATIONS 5 minutes to 24 hours
                      Return Period Rainfalls(mm)
       2yr  5yr  10yr  20yr  30yr  50yr 100yr 150yr 250yr
5min   5.8  7.3   8.4   9.5  10.2  11.1  12.5  13.4  14.5
10min  8.1 10.2  11.7  13.2  14.2  15.5  17.4  18.6  20.3
15min  9.6 12.0  13.8  15.6  16.7  18.2  20.5  21.9  23.8
30min 13.4 16.8  19.2  21.6  23.2  25.3  28.4  30.3  33.0
1hr   18.6 23.3  26.6  30.1  32.2  35.1  39.3  42.0  45.7
2hr   26.0 32.5  37.0  41.8  44.7  48.6  54.5  58.2  63.2
3hr   31.6 39.4  44.9  50.6  54.2  58.9  65.9  70.4  76.4
4hr   36.3 45.2  51.5  58.0  62.1  67.5  75.5  80.6  87.5
6hr   44.0 54.9  62.4  70.3  75.2  81.7  91.4  97.5 105.8
9hr   53.5 66.6  75.7  85.2  91.1  99.0 110.6 118.0 128.0
12hr  61.4 76.4  86.8  97.7 104.4 113.4 126.6 135.1 146.4
15hr  68.4 85.0  96.5 108.6 116.0 126.0 140.7 150.0 162.6
18hr  74.6 92.7 105.3 118.4 126.5 137.3 153.3 163.4 177.1
24hr  80.2 99.2 112.3 125.8 134.2 145.4 162.0 172.4 186.5
```

Figure E.1: Sample output from Point1.R for Delphi Lodge, Co Mayo

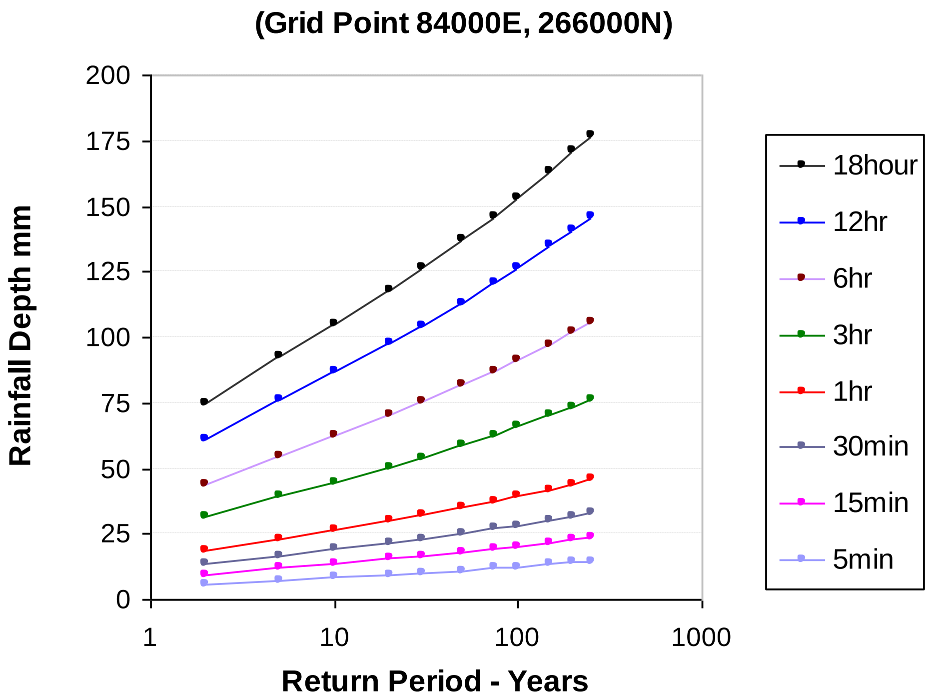

Figure E.2: Rainfall frequency curves for 5 min to 18 hours at Delphi Lodge, Co Mayo

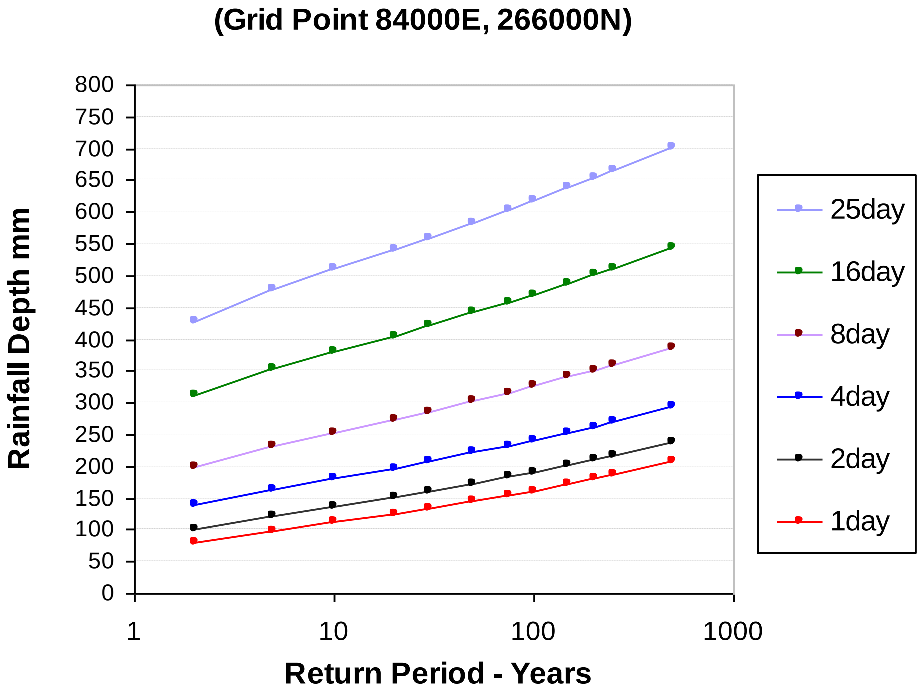

Figure E.3: Rainfall frequency curves for one to 25 days at Delphi Lodge, Co Mayo

E2.3 Use of model outputs

For durations shorter than 24 hours, the DDF model may be used with fair confidence for return periods up to 250 years. For durations of one day or longer, the model may be used with fair confidence for return periods up to 500 years. The programs will produce outputs beyond 250 or 500 years but these should be treated with caution, more especially for the shorter durations. It should be noted that all outputs produced by the computer programs are for sliding durations. Advanced users will note the standard errors detailed in Appendix D.

E3 Interpolation method

The programs interpolate the model parameters to the subject location and then apply the rainfall DDF model. For a general point within a grid box, the four nearest grid points are used in interpolation. For a location on a gridline, only the two nearest grid points are used in interpolation. The weights used to interpolate between the grid-point parameter values are set according to the inverse of the square of their distances from the subject location.

Where the subject location coincides with a 2-km grid point, the parameter values for that grid point are adopted without modification. However, an exception to this last rule was made for sub-daily durations, where the subject grid-point and its four neighbours were used with equal weights of 1/5 each (see Section C3).